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CHARACTERISTICS OF PHOTOGRAPHIC DESENSITIZERS AND DISTORTIONS ON PLATES DUE TO LOCAL DESENSITIZING[1]

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ABSTRACT

The sensitivity of photographic plates can be reduced by bathing in solutions of certain dyes or certain classes of chemicals. The action is of two distinct types: some chemicals act by oxidizing the silver nuclei, others act by catalysis. The first class has the power to destroy the latent image, as well as to reduce the sensitivity, while the second class can affect the sensitivity only. The desensitizing characteristics of copper sulphate, a catalyzing desensitizer, have been studied in detail. It is found to be a general characteristic of moderate desensitizing to increase the contrast of the plate. A theory of this action is outlined. The characteristics of the dye desensitizers are studied and found to be similar to those of copper sulphate. The oxidizing desensitizer, chromic acid, is found to produce a much greater degree of desensitizing, due to its ability to destroy the latent image. On the whole, copper sulphate appears to be the most useful and dependable desensitizer.

In astronomical photography it is frequently necessary to desensitize small areas near the center of the plate, as in parallax work. In this case it is of importance to study the distortions produced within the desensitized area. This has been done in detail for various sizes of spots. In general there is found to be a symmetrical radial movement of the gelatine outward within the wetted area, the movement ceasing abruptly at the boundary, where the gelatine is piled up, forming the well-known water-spot ring. This movement ceases at about 19 mm from the edge of the spot, so that in order to have an area within the spot which shall be free from distortion, the spot must be at least 38 mm in diameter. It is shown that these distortions can be completely eliminated by soaking the entire plate in water immediately after the desensitizing bath, which, however, reduces the degree of desensitizing. The distortion may also be eliminated by desensitizing with chromic acid, applied after the exposure, but this treatment is not recommended. It is concluded that if the desensitized spot is at least two inches in diameter, the factor of safety is sufficient for parallax work of the highest precision.

[1] Communication No. 220 from the Research Laboratory of the Eastman Kodak Company.

The usefulness of a desensitizer in astronomical photography depends upon the preservation of those characteristics of a photographic plate which make for good rendering. The important qualities to be preserved are contrast, resolving-power, and sharpness. In addition there must be uniformity of desensitization and freedom from fog and from graininess. These characteristics depend upon (*a*) selective action of desensitizer on grain-size; (*b*) selective action on grain-sensitivity; (*c*) opacity or turbidity effects, controlling the penetration and scatter of light in the emulsion.

In studies of desensitization, the desensitizer is generally removed from the emulsion before exposure, either by washing or chemical treatment. In this way the sensitivity characteristics of the desensitized grains can be studied without the complication of the presence of the desensitizer either during the process of the formation of the latent image or of its subsequent development. The opinion is widely held that thus restricted the grain-sensitivity may be reduced only to a certain definite minimum, namely, that of the sensitivity of silver bromide crystals, the hypersensitivity conferred on the grains by the sensitivity nuclei alone being removed. Thus, residual sensitivity is roughly one one-hundredth that of the hypersensitized grain. A much greater degree of desensitizing may be secured by allowing the desensitizer to remain in the emulsion. The action in this case, at least for the oxidizing desensitizers, from the viewpoint of the silver-grain nuclear theory, is clear, and of interest. As fast as the developable centers are formed by the action of the light, they are oxidized by the desensitizer present. On the other hand, those sensitivity centers formed in the interior of the grain by the penetrating light remain untouched by the desensitizer. In that case, however, they are equally inaccessible to the developer. Treatment with potassium iodide is found to break the grains apart, as has been shown by Luppo-Cramer and others, allowing access of the desensitizer to the buried centers. This has been proved by the restoration of sensitivity by the iodide treatment, and in addition by its subsequent removal with oxidizer. While it is thus easy to form a fairly complete picture of the mechanism of oxidizing desensitization, when we come to study the action of the desensitizers

of the type of metallic salts, of which copper sulphate, uranium nitrate, and iron and mercury salts are examples, and, in addition, the dye desensitizers, such as phenosafranin and pinacryptol green, the mechanism is by no means clear. In these cases the action is supposed to be catalytic, operative only during the exposure. It is suggestive to note that dyes of certain types confer an additional or hypersensitivity on the grain, while those just mentioned will remove the normal sensitivity. In both cases, exceedingly minute quantities are effective. The analogy, however, does not appear to extend to selective color-effect, except to a very limited extent. On orthochromatic plates, the desensitizing effect in the yellow and green appears to be double that in the blue and violet.

The difference between desensitizers and those substances which destroy the latent image should be noted. Only desensitizers of the oxidizing class can destroy the latent image already formed. The metallic salts and dye-desensitizers act only on the sensitivity. There is thus suggested a fundamental difference between the actions in the two cases. The action of the first class, the oxidizers, is undoubtedly chemical. That of the second class, on the other hand, cannot be chemical in the usual sense, but is undoubtedly catalytic. The action may be pictured as a restraining field of force, preventing the formation of the latent image, but is of such a nature that once the latent image is formed, it is powerless to act.

Characteristics of plates desensitized by copper sulphate; plate used, Eastman 33.—Concentration of desensitizer, 1 per cent. The effects of varying the bathing time, and of washing, are clearly shown in the table on page 340. Development, in MQ Process developer[1] at 70° F. for five minutes, giving average full development. A wide variation in desensitization is exhibited. The effect of washing in water after the desensitizing bath is pronounced, the action being notably reduced. Of equal interest is the effect on gamma or contrast, short bathing (10 sec.) giving a marked increase

[1] Formula:

Elon	1 gr.
Hydrochinon	9
Sodium sulphite	75
Potassium carbonate	25
Potassium bromide	5
Water	1000 cc

compared with the unbathed plate. For the three-minute bath, the contrast is equal to that of the unbathed plate, while prolonged bathing, without subsequent washing, results in a further reduction.

TABLE I

	Speed H and D	Ratio Decrease	Gamma
A. Untreated plate	83.0	....	2.02
B. Bathed 10 sec. in desensitizer	3.1	27	2.54
C. Bathed as in B, washed 5 min. before drying	6.4	13	2.68
D. Bathed 3 min. in desensitizer	1.8	46	2.03
E. Bathed as in D, washed 5 min. before drying	5.2	16	2.40
F. Bathed 60 min. in desensitizer	0.63	132	1.66

The plate-curves corresponding to A, B, D, and F are shown in Fig. 1.

In all cases, washing the plate increases the contrast. In addition to these tests, resolution tests were carried out, which gave an increase of resolving power, of an amount comparable with the rise

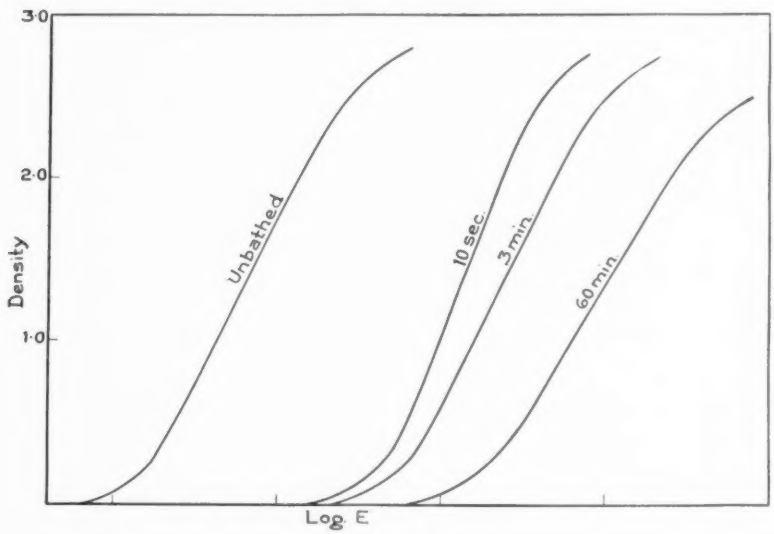

FIG. 1

in gamma. In the case of the plates treated for ten seconds, a notable decrease in fog and graininess was observed. On the whole, therefore, very desirable photographic qualities appear to be

obtained by this particular desensitizing treatment. With longer bathing times or greater concentration of the copper sulphate, especially if the plate is not washed after the desensitizing bath, an increase of fog is obtained. This may be due to the fact that copper is a catalyst for chemical fog.

The increase in contrast on desensitizing is of great theoretical interest in that it is capable of furnishing clues to the mechanism of the light-action on the sensitive grains. Generally considered, the increase in contrast can be explained as a relatively greater desensitizing action on the weaker exposures, from which it would be inferred that the larger and more sensitive grains are desensitized to a greater extent than the smaller. But experiments of Sheppard, Wightman, and Trivelli[1] show that this is not the case. They find that the smaller and presumably the relatively insensitive grains are more strongly affected by the desensitizer. This is by no means opposed to the observed increase in gamma, which can be shown to take place whenever the *dispersion* in grain size and sensitivity is reduced. To see this clearly it is only necessary to compare the uniform-grain, high-contrast process plate with the low-contrast, high-speed plate containing grains of a wide range in size. If we assume that the sensitivity of the grain is located within very minute areas over the surface and in the interior as well, and that large grains have many such spots, and small grains perhaps only one or two, the selective action of the desensitizer on the small grains is not hard to visualize. For, assuming a random or probability action of the desensitizer, more small grains, with one sensitivity center say, would be put out of commission, so to speak, than large grains, having numerous sensitivity centers. The action is the inverse of that of the formation of the latent image. In that case the larger grains are affected first, or rendered developable, because they have individually a large number of sensitivity centers, and it is only necessary for one center to be affected by the light-action in order to make the whole grain developable. In the corresponding case for desensitizing, *all* of the centers must be affected before the grain can be considered to be insensitive. The chance of this happening is very remote. In the case of small grains, with one or

[1] "Studies in Photographic Sensitivity," *Journal of the Franklin Institute*, **196**, 35, 1923.

two sensitivity centers only, the chance of total disability is very much increased. This theory of the action is supported by the numerical results of Sheppard, Wightman, and Trivelli (*loc. cit.*), who find that for longer exposures the selective action is decreased. The complete investigation of this problem mathematically should give interesting results, but is outside the scope of the present paper.

It is to be noted that the desensitized grains are not permanently or completely desensitized, for heavy exposures will be found to render them developable. This is explained by assuming that the grains have lost merely their hypersensitivity, or that conferred on them by the sensitivity spots. There remains the residual or basic sensitivity of silver bromide which does not appear to be affected by the desensitizer.

A further series of experiments was made, using a higher concentration (5 per cent) of copper sulphate. The experiments were planned to show the effect of washing the bathed plate respectively before and after exposure. The bathing time was three minutes; washing time, fifteen minutes. Results were as follows, plate and development conditions being the same as in Table I.

TABLE II

	Ratio of Speed Decrease	Gamma	Fog
A. Untreated plate..........	....	1.84	0.12
B. Bathed; washed before exposure..........	19	2.36	.17
C. Bathed; washed before development..........	120	1.24	.06
D. Bathed; not washed..........	320	1.14	0.43

The *H* and *D* curves are plotted in Fig. 2.

The greatest reduction in speed is seen to be for treatment D, i.e., plate not washed. But contrast is low and fog excessive. If high contrast is not needed, treatment C is to be recommended, i.e., washing before development, on account of the low value of fog and high degree of desensitizing. At first sight it is difficult to understand the low value of gamma for treatments C and D in the table, if one accepts the theory previously outlined (p. 341). But it will be noticed that in both cases the reduction of sensitivity has

been very great, the supersensitivity being largely destroyed. The sensitivity of the silver bromide crystals accordingly comes into play and controls the contrast relation of the plate formerly controlled by the sensitivity centers.

Desensitizing characteristics of phenosafranin and pinacryptol green.—These dyes, added to the developing bath, have proved useful in practical and commercial photography, allowing the developing plate to be examined under comparatively strong light. They are desensitizers only and thus do not affect the latent image.

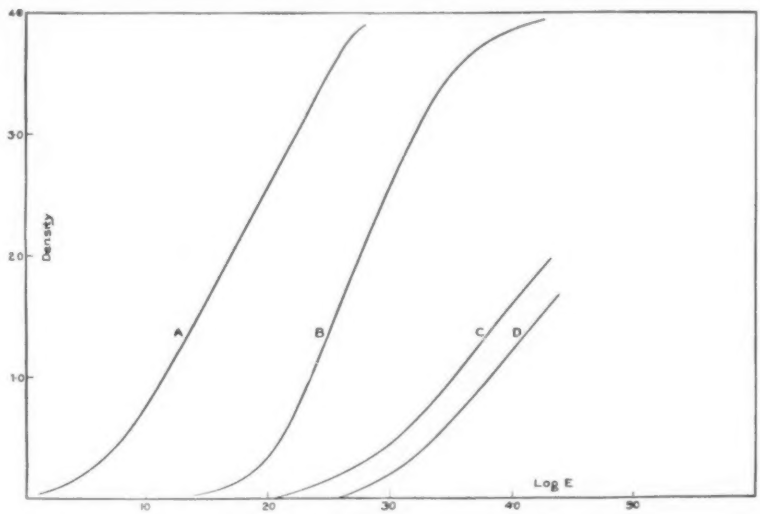

FIG. 2

Pinacryptol green is to be preferred, on account of its non-staining qualities. The characteristics were found to be quite similar to those of copper sulphate. Bathing in a 1 : 100,000 solution for one minute was found to diminish speed ten times and to increase contrast 10 per cent. A concentration of 1 : 25,000, which is that usually employed, diminished the speed forty times and the contrast 10 per cent. The contrast thus rises and falls according to the concentration in a manner similar to that for copper sulphate. The desensitizing power of the two dyes is practically the same.

Chromic acid as a desensitizer.—On account of its strong oxidizing power, chromic acid will destroy the latent image as well as act as a

desensitizer. Its desensitizing power is very great if the excess is not washed out of the emulsion before exposure. Thus, bathing for three minutes in a 0.5 per cent solution without subsequent washing may reduce the sensitivity ten thousand times. With a strength one-third of this, the reduction is but one hundred and twenty times. Its desensitizing characteristics have not been as completely investigated by the writer as in the case of copper sulphate. In Figure 3, *A*, the *H* and *D* curve of an untreated plate (Eastman 33) is shown. *B* is the curve for a plate bathed for two

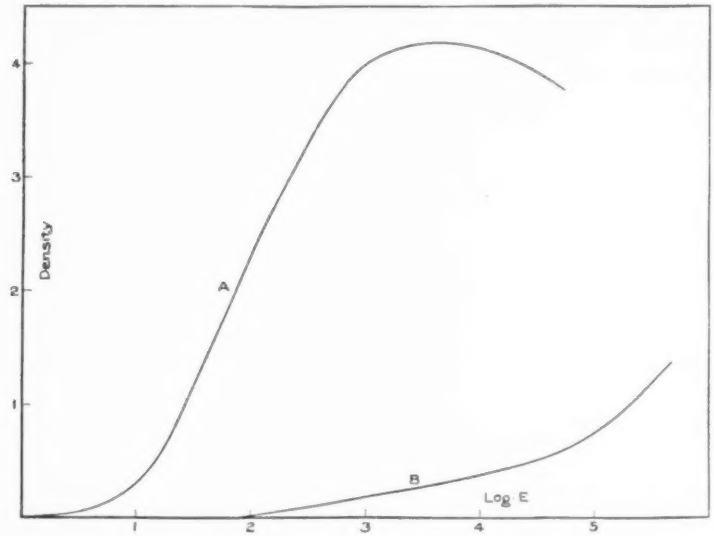

Fig. 3

minutes in 0.5 chromic acid after exposure. The bath was preceded and followed by a one-minute bath in water. It is seen that the sensitometric characteristics are enormously changed by the treatment. Virtually two straight line portions are obtained—one of low contrast and of very long range, followed by a straight line of much greater slope extending far into the reversal region of the untreated plate. It should be mentioned that very high illuminations were used for the strongest exposures, print-out images being obtained. The complete series of curves for varying concentrations of the developer would be of interest, but does not appear to be of any great importance for our purpose.

We are apt to look upon the overexposure region, or overexposed plate, as valueless. Even although the densities throughout the overexposure may be too high for measurement, and may indeed show reversal, it is apparent from curves *A* and *B* of Figure 3 that by suitable treatment of the plate after exposure and before development densities are toned down and contrasts preserved to an extent sufficient in many cases to give results of value. In order to show these in a practical case, the writer exposed two films on an outdoor subject—one of normal exposure, the second one hundred and fifty times normal. The latter, before development, was bathed for one minute in 0.5 per cent chromic acid, then three minutes in water. The two negatives were nearly identical. The method is worth considering in some branches of astronomical photography where the range of light-intensities is too great to be handled on the ordinary photographic plate, speed being of secondary consideration.[1]

DISTORTIONS ON PHOTOGRAPHIC PLATES WITHIN SMALL DESENSITIZED AREAS

Spotting with water.—A small drop of water was placed on a dried photographic plate so that its center coincided with a line of artificial star-images which had previously been photographed on the plate. After allowing sufficient time for the water to soak in, the surplus was removed and the relative distances of the star-images which were contained within the "spot" were measured on the comparator and compared with the mean position of all the images, both inside and outside the spot. The results are shown in column B in Table III. Column A contains the measurements before spotting. The plate was finally soaked in water for one hour, dried, and remeasured, with results shown in column C. The fifth column contains the distortions due to spotting, which are seen to be confined entirely within the wet area. The sixth column shows that the final soaking of the entire plate has reduced the distortions considerably. Results are shown graphically in Figure 4, which clearly indicates the nature of the phenomenon taking place, a symmetrical expansion of the film from the center of the spotted area outward. The greatest differential movement is at the center.

[1] J. Sterry, *Photographic Journal*, **30**, 171, 1907.

At this point it is seen that the expansion of the film is 47 μ in a distance of 0.7 mm, or 7 per cent. The points *A* and *H*, which are only 0.2 mm outside the edge of the spot, show no displacement whatever. The gelatin inside the spot, moving outward, is accordingly compressed against the spot's edge beyond which it cannot go. This accounts for the ring usually seen at the edge of a water-spot on a photographic plate. The spot and ring can generally be removed by prolonged soaking in water. The percentage compression of the gelatin at the edge of the spot cannot be determined from the curve.

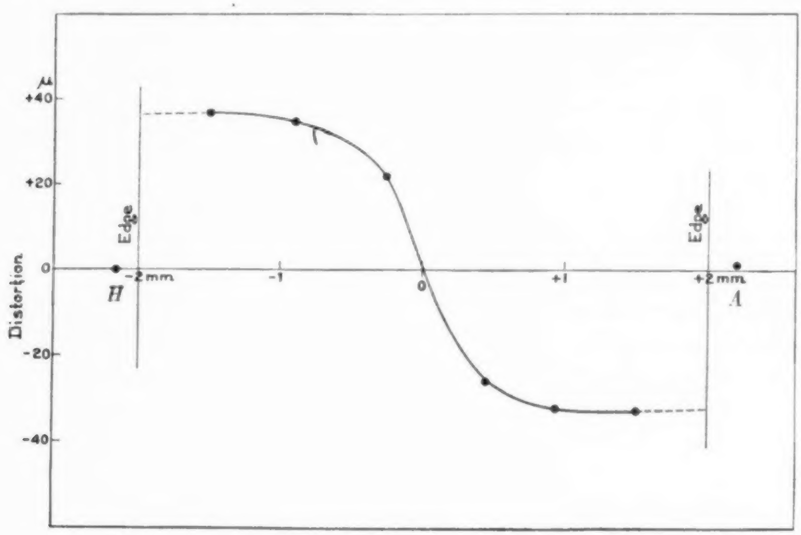

FIG. 4

Effect of hardener in the fixing bath.—The results of Table III and Figure 4 were obtained on plates which had not been hardened. The effect of hardener in the fixing bath is shown in Table IV, which contains the results of measurement on a hardened plate, treated in all other respects similarly to the preceding.

Comparing Tables III and IV, it is seen that hardening the plate has reduced the distortion to one-third, and in addition the distortions appear to be completely removed by soaking in water (column C–A). The advantage of hardening the gelatin in the matter of reducing the distorting effects of spotting is manifest.

In the previous experiment it has been shown that when a plate is spotted with water, the effect is an outward symmetrical radial

TABLE III

Image	Mean (mm)			Difference μ	
	A	B	C	B-A	C-A
1	+3.671	+3.671	+3.672	0	+1
2	2.989	2.987	2.985	−2	−4
3	2.341	2.339	2.340	−2	−1
4	1.666	1.666	1.666	0	0
5	1.412	1.412	1.415	0	+3
Wet line	1.26				
6	0.768	0.805	0.775	+37	7
7	+ .086	+ .120	+ .092	34	6
8	−0.564	−0.542	−0.562	+22	+2
9	1.236	1.262	1.241	−26	−5
10	1.719	1.751	1.729	32	10
11	2.368	2.401	2.374	−33	−6
Wet line	2.83				
12	3.051	3.050	3.050	+1	+1
13	3.698	3.698	3.696	0	2
14	4.372	4.371	4.370	+1	2
15	4.865	4.866	4.865	−1	0
16	−5.510	−5.510	−5.509	0	+1

TABLE IV

Image	Mean (mm)			Difference (μ)	
	A	B	C	B-A	C-A
1	+3.690	+3.691	+3.690	+1	0
2	3.007	3.007	3.004	0	−3
3	2.360	2.359	2.360	−1	0
4	1.685	1.684	1.686	1	+1
5	1.292	1.290	1.291	−2	−1
Wet line	0.87				
6	+ .647	+0.653	+0.647	+6	0
7	− .038	− .026	− .037	+12	+1
8	0.685	0.684	0.685	−1	0
9	1.360	1.372	1.360	12	0
10	1.782	1.795	1.782	−13	0
Wet line	2.01				
11	2.426	2.425	2.424	+1	+2
12	3.110	3.111	3.112	−1	−2
13	3.757	3.758	3.757	1	0
14	4.431	4.431	4.431	0	0
15	−4.815	−4.816	−4.815	−1	0

displacement of the gelatin within the spot. It is to be expected, therefore, that if a plate is spotted before being exposed, the gelatin

within the spot will be in displaced condition corresponding to the large distortion found on unhardened plates. If such strain or displacement persists after exposure, through the subsequent operations of development, fixing, washing, and drying, it is evident that measures of relative position, such as are made on astronomical photographs, will not be affected by any error. But if the principle previously formulated[1] governs the action under such conditions, namely, that gelatin tends to return to its original state if the final treatment to which it has been subject is normal, in spite of abnormal intermediate processes of treatment, it is to be expected that in the case under consideration the plate will show opposite displacements to those found in Table III, i.e., there will be an apparent contraction of the gelatin instead of an expansion as in the table. To what extent this is the case is the object of the following series of measurements.

In cases occurring in astronomy where desensitized spots are found useful, the spotted areas are generally much larger than those in Tables III and IV. For example, when used in connection with determining parallaxes of the very bright stars, 2 inches has been found a convenient size. They are most conveniently made by means of a template, the desensitizer being applied with a wad of cotton. It is found that with care a uniformly desensitized area, with sharp boundaries, can be secured. Instead of spotting with water, as in the previous experiments, a one per cent solution of copper sulphate was used, giving a speed reduction of fifty times.

There are two important facts to determine in the case of a desensitized spotted area formed as above: (*a*) amount of distortion or displacement, and its variation over the spotted area; (*b*) the effect of spotting on the probable error of position of a star-image located within the area, and, in addition, the variation of probable error over the area. A certain amount of information under (*a*) has been already obtained (Tables III and IV). It is important, however, to make the determination for larger spots, made under conditions holding in practical work. Under (*b*), no information has been obtained, although it is equally important, for even if it is

[1] *Astrophysical Journal*, "Image Contraction and Distortion on Photographic Plates," **52**, 108, 1920.

shown that in the center, for example, of a spotted area, there are no systematic distortions, there remains the possibility of accidental distortions, due to the somewhat rude treatment of the area in question.

The plan of the method is as follows: A reseau, consisting of fine parallel lines spaced a millimeter apart, is printed by contact across the center of the desensitized area. This was done on seven desensitized plates, and also on seven control plates which had not been spotted. Each plate was measured twice, direct and reversed, but the spotted plates were measured four times. Measurement was always started at the same reading on the comparator so that screw and bearing errors are eliminated. Since the displacements can be assumed, from the previous work, to be a function of the distance from the center of the spot, it will be sufficient to form from the comparator readings the distances Δ between lines of the reseau which are symmetrically situated with respect to the center of the spot. The reseau was centered on each plate so that symmetrical lines are always the same reseau lines. By comparison with the mean values of Δ on the control plates, the distortions are directly obtained, and by intercomparison of the seven plates of each class the dispersions, which represent the instability of the gelatin as noted above, can be obtained. The mean results are tabulated in Table V for each pair of lines, on the control and on the spotted plates, respectively. It is seen that the distances on the spotted plates have contracted, as expected. The lines near both ends of the scale are necessarily overexposed in the case of the spotted plates, so that measures at these points have little significance. The smooth curve obtained from the measures of Table V is plotted in Figure 5 and the probable errors in Figure 6. The conclusion is drawn from the table and figures that only a small area in the center of the spot, in this case about 8 mm in diameter, is free from displacement. In addition, from Figure 6, it is noted that for this restricted central region, the probable error is no greater than on the control plates. It may be concluded that for a spot of the size considered, namely, 46 mm in diameter, measurements can be made as precisely as on an untreated plate, provided one keeps within approximately 4 mm of the center of the spotted area. This means

that distortions are sensible up to 19 mm from the edge of the spot. If a larger area free from distortion is desired, the size of the spot must be increased. The diameter of the distortion-free area will be roughly the diameter of the spot decreased by 38 mm.

From Figure 6 it will be noticed that the probable errors increase with the distortion, from which it may be concluded that there is

TABLE V

LINES	Δ				$\Delta n - \Delta s$
	Δn	*p.e.*	Δs	*p.e.*	
	mm		mm		
24–26......	2.0033	±15	2.0025	±17	+.0008
23–27......	3.9600	17	3.9604	18	−.0004
22–28......	5.9453	17	5.9429	18	+.0024
21–29......	7.9120	19	7.9114	22	+.0006
20–30......	9.8913	18	9.8914	16	−.0001
19–31......	11.8636	13	11.8644	28	−.0008
18–32......	13.8314	13	13.8297	21	+.0017
17–33......	15.8001	17	15.7997	27	+.0004
16–34......	17.8144	11	17.8144	29	.0000
15–35......	19.8076	23	19.8044	20	+.0032
14–36......	21.8084	19	21.8055	25	+.0029
13–37......	23.7763	8	23.7727	32	+.0036
12–38......	25.7321	26	25.7277	33	+.0044
11–39......	27.7084	18	27.7012	30	+.0072
10–40......	29.6827	11	29.6773	35	+.0054
9–41......	31.6763	19	31.6707	50	+.0056
8–42......	33.6739	24	33.6682	47	+.0057
7–43......	35.6361	12	35.6256	38	+.0105
6–44......	37.6084	21	37.5985	18	+.0099
5–45......	39.5647	19	39.5537	41	+.0110
4–46......	41.5734	21	41.5575	98	+.0159
3–47......	43.5629	19	43.5518	67	+.0111
2–48......	45.5390	21	45.5244	69	+.0146
1–49......	47.5484	15	47.5424	72	+.0060
0–50......	49.4580	19	49.4593	36	−.0013

Unity of *p.e.* is 0.0001 mm. The *p.e.*'s refer to a single distance and not to the mean value tabulated.

no constancy to the distortions, they varying irregularly from plate to plate.

If the plate is soaked in water directly after the desensitizing bath, the distortion should be eliminated. In this case, however, the degree of desensitization is reduced, a threefold decrease being noted for five minutes' washing. In addition there is an objectionable spreading of the desensitizer in an irregular manner beyond the original area. For certain lines of work, however, these may

not be serious objections compared with the advantage of complete elimination of distortions.

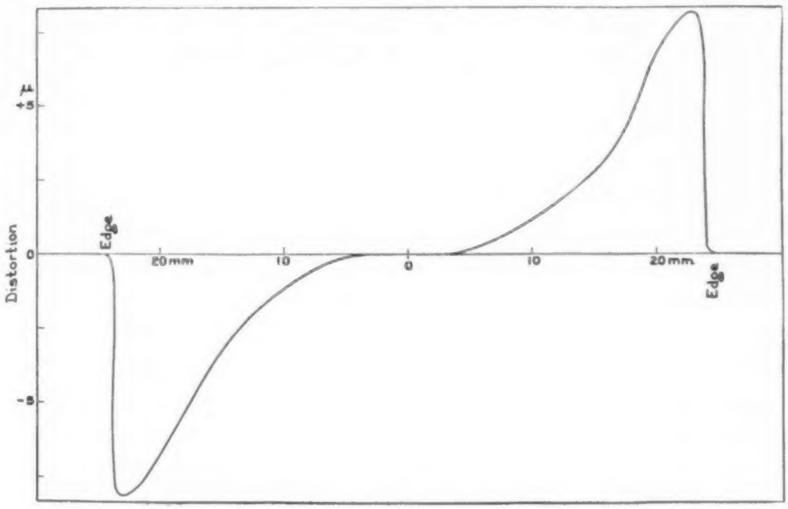

Fig. 5

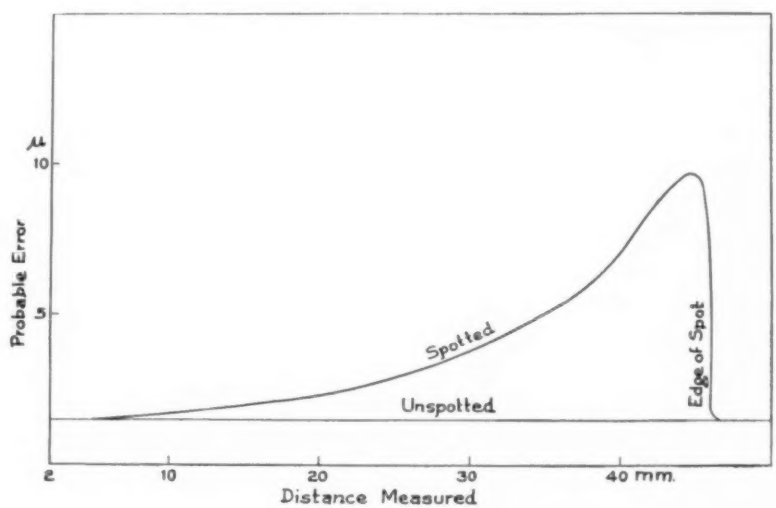

Fig. 6

Distortions may be eliminated in still another manner, making use of the property of chromic acid of partially destroying the latent

image. The procedure would be to bathe the plate after exposure in chromic acid, wash, then develop. In order to find out to what extent the distortions are actually eliminated by this treatment, a plate was exposed to the same scale test object used previously, after which the central portion over a circular area 20 mm in diameter was sponged for one minute with a 0.5 per cent solution of chromic acid, using a template as previously. The plate was immediately soaked in water for two minutes, developed and measured as before. It will not be necessary to give details. As had been anticipated, no systematic distortion was observed. The average residual for twenty lines, without regard to sign, was 0.0028 mm; taken with regard to sign, +0.0005 mm. While the distortion has been thus eliminated, the use of chromic acid, in work of precision, is not to be recommended. The reason for this is apparent from curve *B*, Figure 3. The contrast or gamma for plates treated with chromic acid is seen to be so low that sharpness of outline of star-images, which depends to a great extent on gamma, is very much reduced.

RESEARCH LABORATORY
EASTMAN KODAK COMPANY
October 1, 1925

ANALYSIS OF RADIAL VELOCITIES OF GLOBULAR CLUSTERS AND NON-GALACTIC NEBULAE[1]

BY GUSTAF STRÖMBERG

ABSTRACT

List of radial velocities of globular clusters and non-galactic nebulae.—Table I contains a list of all measured radial velocities of globular clusters and non-galactic nebulae, the great majority of the determinations being by Slipher.

Solar motion.—The sun's motion relative to the two classes of objects studied is given in Table II in rectangular equatorial co-ordinates ($-\xi$, $-\eta$, $-\zeta$) and in polar co-ordinates (A_0, D_0, V_0). The value of the sun's motion relative to the nebulae is 344 km/sec. toward $A_0=305°$, $D_0=+56°$ and, relative to the clusters, 329 km/sec. in the direction $A_0=320°$, $D_0=+65°$.

Curvature of space-time.—On the basis of De Sitter's theory of curvature of space-time, we should expect to find for very distant objects a shift of the spectral lines toward the red end of the spectrum. According to Silberstein, we may expect a shift either toward the red or the violet. The correlation-coefficients between distance and radial velocities give no clear evidence of curvature in either De Sitter's or Silberstein's sense, at least up to the limit of distance studied. The only definite correlation is one between radial velocity and position in the sky, indicating a solar motion of 300 or 400 km/sec. in about the same direction for both classes of objects.

The determination of radial velocities of globular clusters and of non-galactic nebulae is very difficult on account of the faintness of the objects and the absence, in general, of bright lines in their spectra; but through the perseverance of Professor V. M. Slipher, a fairly large number of such velocities has been derived. Two reasons prompted the writer to study these velocities. One was the large solar velocity found from these objects, which, in connection with the asymmetry of stellar motions, indicated that a fundamental reference system could be defined by them. The second reason was the desirability of ascertaining whether the velocities give any evidence of a curvature of space-time.[2] Through the courtesy of Professor Slipher it has been possible to make use of his radial-velocity determinations up to a recent date.

In Table I are collected the data on which the computations are based. Slipher's determinations are given without references; for

[1] *Contributions from the Mount Wilson Observatory*, No. 292.

[2] After this paper was ready for printing, Lundmark's article, "The Determination of the Curvature of Space-Time in De Sitter's World," appeared in *Monthly Notices*, **84**, 747, 1924. Lundmark's conclusions as to space-curvature are about the same as those found in this study.

TABLE I

RADIAL VELOCITIES OF GLOBULAR CLUSTERS AND NON-GALACTIC NEBULAE

N.G.C.	α (1900)	δ (1900)	V		
			Slipher	Others	Mean
			km/sec.	km/sec.	km/sec.
221	$0^h 37^m.2$	+40° 19′	− 300	...	...
224	0 37	+40 43	− 300	− 320*	−315
278	0 46.4	+47 1	+ 650::	...	...
404	1 3.9	+35 11	− 25	...	...
584	1 26.3	− 7 23	+1800:	...	...
598	1 28.2	+30 9	− 260	− 70†	− 70
936	2 22.5	− 1 36	+1300	...	...
1023	2 34.1	+38 38	+ 300	...	...
1068	2 37.6	− 0 26	+1120	+ 814‡	+ 916
1700	4 52.0	− 5 1	...	+ 800§	...
2681	8 46.4	+51 41	...	+ 700‖	...
2683	8 46.5	+33 48	+ 400	...	...
2841	9 15.1	+51 24	+ 600:	...	...
3031	9 47.4	+69 32	− 30	...	...
3034	9 47.6	+70 10	+ 290	...	...
3115	10 0.3	− 7 14	+ 600	...	...
3368	10 41.5	+12 21	+ 940	...	...
3379	10 42.6	+13 7	+ 780	+ 845§	+ 812
3489	10 55.0	+14 26	+ 600::	...	...
3521	11 0.7	+ 0 30	+ 730	...	...
3623	11 13.7	+13 38	+ 800	...	...
3627	11 15.0	+13 32	+ 650	...	...
4111	12 2.0	+43 37	+ 800:	...	...
4151	12 5.5	+39 58	+ 980	+ 940¶	+ 950
4214	12 10.6	+36 53	+ 300	...	...
4258	12 14.0	+47 52	+ 500	...	...
4382	12 20.3	+18 45	+ 500:	...	...
4449	12 23.3	+44 39	+ 200	...	...
4472	12 24.7	+ 8 33	+ 850	...	...
4486	12 25.6	+12 57	+ 800::	...	...
4526	12 29.0	+ 8 15	+ 580	...	...
4565	12 31.4	+26 32	+1100:	...	...
4594	12 34.8	−11 4	+1100	+1180**	+1140
4649	12 38.6	+12 6	+1090	...	...
4736	12 46.2	+41 40	+ 290	...	...
4826	12 51.8	+22 14	+ 150	...	...
5005	13 6.3	+37 36	+ 900	...	...
5055	13 11.3	+42 34	+ 450	...	...
5194	13 25.7	+47 43	+ 270	...	...
5195	13 25.7	+47 43	+ 240:	...	...
5236	13 31.4	−29 21	+ 500:	...	...
5866	15 3.7	+56 9	+ 650	...	...
7331	22 32.5	+33 54	+ 500	...	...
Magellanic	5 27	−68	...	+ 287††	...
clouds	1 6	−73 44	...	+ 168††	...
1851	5 10.8	−40 9	+ 315	...	...
1904	5 20.1	−24 37	+ 235	...	...
5024	13 8.0	+18 42	− 170	− 200‡‡	− 180
5272	15 37.6	+28 53	− 125	− 140‡‡	130

TABLE I—*Continued*

N.G.C.	α (1900)	δ (1900)	*V* Slipher	*V* Others	*V* Mean
			km/sec.	km/sec.	km/sec.
5904	15^h13^m5	$+2°27'$	+10	....	....
6093	16 11.1	−22 41	+70	....	....
6205	16 38.1	+36 39	−300	−230§§	−265
6218	16 42.0	−1 46	....	+160‡‡	....
6229	16 44.2	+47 42	−100:	....	....
6266	16 54.8	−29 58	+50	....	....
6273	16 56.4	−26 7	+30	....	....
6333	17 13.3	−18 25	+225	....	....
6341	17 14.1	+43 15	−160	....	....
6626	18 18.4	−24 55	0	....	....
6934	20 29.3	+7 4	−350:	....	....
7078	21 25.2	+11 44	−95	−93‡‡	−94
7089	21 28.3	−1 16	−10	....	....
7099	21 34.7	−23 38	−125	....	....

* Mean of several determinations.

† Slipher has measured a knot north following the nucleus, whereas the Mount Wilson determination is based on the spectrum of the nucleus; *A.S.P. Pubs.*, **28**, 33, 1916.

‡ Mount Wilson measures give $V=+765$; *A.S.P.*, **27**, 133, 1915; and Lick Observatory gives $V=+864$; *L.O. Pubs.*, **13**, 88, 1918.

§ Mount Wilson, *A.S.P. Pubs.*, **30**, 255, 1918.

‖ *Ibid.*, **34**, 222, 1922.

¶ *L.O. Pubs.*, **13**, 122, 1918.

** *Mount Wilson Comm.*, No. 32; *Proc. Nat. Acad. of Sciences*, **2**, 517, 1916.

†† *L.O. Pubs.*, **13**, 168, 1918.

‡‡ Sanford, unpublished.

§§ Mean of 4 stars, measured by Adams, Joy, and Humason.

other observers, references are given in footnotes. In the list 43 nebulae stand first, most of which have spiral structure, although several are irregular or globular. Then follow the Magellanic clouds and 18 globular clusters. The probable error of the radial velocities for the nebulae is probably about 50 km/sec. and for the clusters about 25 km/sec. The more uncertain values are marked by one or two colons.

THE SOLAR MOTION

The elements for the solar motion were first determined. The equations of condition used were of the following form:

$$\xi \cos \alpha \cos \delta+\eta \sin \alpha \cos \delta+\zeta \sin \delta+K=V \qquad (1)$$

where ξ, η, and ζ are the rectangular equatorial components of the group-motion as referred to the sun. The co-ordinates of the sun's

apex (A_0, D_0) and its velocity (V_0) relative to the groups of objects studied are determined by the equations[1]

$$\begin{aligned} V_0 \cos A_0 \cos D_0 &= -\xi \\ V_0 \sin A_0 \cos D_0 &= -\eta \\ V_0 \sin D_0 &= -\zeta\,. \end{aligned} \tag{2}$$

Several solutions were made to test the effect of omissions, especially that of the large velocity +1800 km/sec. of N.G.C. 584. Whether we ought to include the Magellanic clouds among the non-galactic nebulae is doubtful, but solutions were made including these objects, both with and without K-terms. The results of the different solutions are given in Table II. The second, fifth, and seventh solutions for the nebulae were made after the radial velocity of N.G.C. 584 had been omitted. This omission seems to be justified on account of the small number of objects used. In the last solution for nebulae and clusters, 13 and 8 groups, respectively, were formed by combining objects near each other in the sky, the different groups being given equal weights. The probable errors are given below the numbers to which they belong. The bracketed probable errors for the right ascension of the sun's apex are reduced to a great circle. The quantity σ is the mean square residual velocity.

The first solution for the nebulae is identical with Lundmark's[2] values, which are based on practically the same material. His values are:

$$\xi = -\ \ 96 \pm 48; \qquad \eta = +138 \pm 108 \text{ km/sec.}$$
$$\zeta = -629 \pm 90; \qquad K = +793 \pm \ \ 59 \text{ km/sec.}$$

By omitting N.G.C. 584, the velocity of the sun decreases from 600 to 500, or to even less than 400 km/sec. The last solution for the nebulae is probably the best and agrees closely with that derived from the globular clusters. The position of the sun's apex as determined from the nebulae and clusters can be fixed at $\alpha = 315°$, $\delta = +62°$, with an uncertainty of about 10° in each co-ordinate. The velocity as determined from nebulae and clusters is 350 and 300 km/sec., respectively, the latter having the higher weight.

[1] Several writers omit the negative signs of the right-hand side, and instead give V_0 a negative sign. As the direction of the sun's motion is given by A_0 and D_0, it seems to the writer that V_0 must be used without sign.

[2] *Observatory*, **47**, 279, 1924.

TABLE II

SUN'S MOTION RELATIVE TO NON-GALACTIC NEBULAE AND GLOBULAR CLUSTERS

No.	ξ	η	ζ	K	A_0	D_0	V_0	σ	Remarks
					Non-galactic Nebulae				
	km/sec.	km/sec.	km/sec.	km/sec.			km/sec.	km/sec.	
43...	−98 ±62	+112 ±140	−610 ±115	+785 ±75	311° (±9)	+76° ±10	628 ±115	345	Mag. clouds omitted.
42*...	−196 ±58	+162 ±127	−450 ±106	+647 ±72	320 (±12)	+61 ±11	517 ±103	302	Mag. clouds omitted.
45...	−170 ±66	+231 ±150	−246 ±94	+565 ±65	+306 (±13)	+41 ±17	378 ±121	380	Mag. clouds included with K-term.
45...	−109 ±60	+121 ±120	−506 ±97	+737 ±69	312 (±10)	+72 ±10	531 ±97	344	Mag. clouds included without K-term.
44*...	−205 ±56	+162 ±121	−386 ±89	616 ±65	321 (±12)	+56 ±11	468 ±89	308	Mag. clouds included without K-term.
45...	−65 ±102	+142 ±188	−332 ±144	+668 ±94	295 (±19)	+65 ±26	367 ±120	....	13 groups with equal weights. Mag. clouds included without K-term.
44*...	−111 ±89	+160 ±160	−284 ±124	+622 ±82	305 (±22)	+56 ±21	344 ±124	....	13 groups with equal weights. Mag. clouds included without K-term.
					Globular Clusters				
18...	−79 ±42	+108 ±41	−253 ±50	+25 ±30	306° (±8)	+62° ±9	286 ±49	113	
18...	−81 ±41	+84 ±27	−261 ±47	0	314 (±7)	+66 ±7	286 ±45	114	$K=0$.
18...	−104 ±45	+108 ±43	−288 ±60	+21 ±32	314 (±8)	+63 ±8	325 ±58	......	8 groups of equal weight.
18...	−106 ±41	+88 ±28	−299 ±53	0	320 (±6)	+65 ±7	329 ±50	......	8 groups of equal weight, $K=0$.

* N.G.C. 584 omitted.

The reality of this large solar motion can perhaps best be judged from the correlation diagram in Figure 1. This shows the correlation between observed radial motion and cos λ, where λ is the angular distance from the sun's antapex, which is assumed to be the same for the two classes of objects. If we omit N.G.C. 584, the correlation coefficients between V and cos λ for nebulae and clusters are $+0.54$ and $+0.73$, respectively, and the equations for the regression-lines of V on cos λ are, respectively,

$$V=452 \cos \lambda+648 \qquad \text{and} \qquad V=295 \cos \lambda+4 \,.$$

The Magellanic clouds are indicated by crosses in the diagram for the clusters, but are not used in the derivation of the regression-

lines. All indications are that we are justified in our assumption of a vanishing value for the K-term for the clouds. The dotted horizontal line corresponds to $K=+616$ km.

The assumption of a constant K-term for the nebulae is probably only approximately correct; the K-term is probably much larger for N.G.C. 584 and smaller for the Andromeda nebula than for the rest of the objects. A variable correction to the wave-lengths would account for the fact that the dispersion in radial velocity is larger

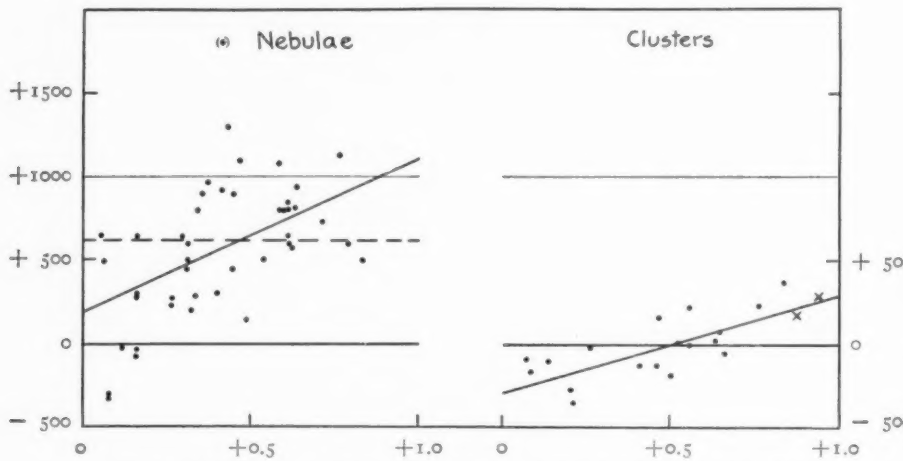

FIG. 1.—Diagram showing correlation between radial velocities (ordinates) and cos λ (abscissae) for nebulae and clusters.

for the nebulae than for the clusters. The vanishing value of the K-term for the clusters can be seen directly from the second scatter-diagram in Figure 1.

The direction of the sun's motion is very nearly the same as that derived from the asymmetry in the distribution of stellar velocities on the assumption of the existence of a velocity-restriction in a fundamental reference-frame.[1] The direction of the sun's motion as derived from the asymmetrical velocity-distribution of stars in our neighborhood is $\alpha=327°$, $\delta=+61°$, and its velocity relative to the stars of highest velocity-dispersion is about 280 km/sec.

[1] *Mt. Wilson Contr.*, No. 275; *Astrophysical Journal*, **59**, 228, 1924.

RADIAL VELOCITIES AND CURVATURE OF SPACE

In Professor de Sitter's[1] theory of space-time of cosmical extent, a constant radius of curvature has been postulated, the expression for the line-element ds in space-time being given by the equation

$$ds^2 = -R^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)] + \cos^2\chi\, dt^2\,. \qquad (3)$$

This expression is the equation of a hypersphere in five dimensions, the distance along a geodesic to a point being given by $\rho = R\chi$, where R is the curvature-invariant. Equation (3) differs from Einstein's expression for an elliptical (or cylindrical) space-time by the factor $\cos\chi$ for dt. In Einstein's world, the time-dimension is not curved, and the difference in curvature between space and time makes it possible to single out a fundamental space in which all the matter in the universe is at rest, and an absolute time, which is the time for an observer at rest in this absolute space. De Sitter's world, on the contrary, is perfectly isotropic.

When the distance to a star is very large, the variable $\chi = \rho/R$ may differ appreciably from zero, and when $\chi = \pi/2$, the interval ds becomes independent of dt. This is interpreted as meaning that the time appears to slow down at large distances, and we should accordingly expect an apparent slowing down of the atomic clock, i.e., a shift of the spectral lines to the red end of the spectrum. Such an effect would produce a fictitious positive radial velocity.

In Silberstein's[2] opinion, if we make the assumption that the motions are so adjusted that they are necessarily small when the object comes near the sun, we may expect a shift either to the red or to the violet.

The present collection of radial velocities can be used to ascertain whether the measured radial velocities are dependent upon the distances of the objects. The velocities of some globular clusters have been used for this purpose by Silberstein, but the result cannot be regarded as conclusive. One thing is obvious, however. If the nebulae studied are at about the same mean distance, or if they are nearer than the globular clusters, we cannot regard the large positive K-term as a De Sitter effect, as the K-term for the clusters must be

[1] *Monthly Notices*, **78**, 3, 1917.

[2] *Monthly Notices*, **84**, 363, 1924; *Phil. Mag.*, **47**, 907, 1924; **48**, 619, 1924.

very small. On the basis of De Sitter's theory, we may expect an increase of radial velocity (V) with distance; on Silberstein's theory, an increase of radial velocity, without regard to sign ($|V|$), with increasing distance from the sun.

The writer has derived coefficients of correlation between distance, on the one hand, and radial velocity with and without sign (V and $|V|$), on the other, and similar coefficients after the velocities have been corrected for a solar motion of 300 km/sec. (V' and $|V'|$). The data for these correlations are collected in Table III. The dis-

TABLE III

CORRELATION TABLE

Objects	No.	Variates	r	Regression-Lines
Clusters..........	18......	V, ρ	-0.06	$V=-1.338\,\rho+2$; $\rho=-0.0029\,V+19$
	18......	$\lvert V\rvert, \rho$	$+0.26$	$\lvert V\rvert=+3.281\,\rho+77$; $\rho=+0.0200\,\lvert V\rvert+16$
	18......	V', ρ	$+0.02$	$V'=+0.326\,\rho+18$; $\rho=+0.0016\,V'+19$
	18......	$\lvert V'\rvert, \rho$	$+0.33$	$\lvert V'\rvert=+2.770\,\rho+43$; $\rho=+0.0403\,\lvert V'\rvert+15$
	18......	$V, \cos\lambda$	$+0.73$	$V=+295\cos\lambda+4$; $\cos\lambda=+0.001820\,V-0.05$
Nebulae..........	29......	V, ρ'	$+0.28$	$V=+1.323\,\rho'+545$; $\rho'=+0.0596\,V+53$
	29......	V', ρ'	$+0.23$	$V'=+0.960\,\rho'+639$; $\rho'=+0.0566\,V'+52$
	41......	$V, \cos\lambda$	$+0.54$	$V=+452\cos\lambda+648$; $\cos\lambda=+0.000647\,V-0.54$

tances for the globular clusters (ρ in kiloparsecs) are Shapley's[1] determinations. For the nebulae we do not know the distances, but we can probably get some rough indication of their relative distances by assuming that they have nearly the same total brightness. As a measure of the distance I have consequently used the quantity $\rho'=10^{0.2\,m}$, where m is the total apparent brightness taken from Wirtz's[2] study. The correlation coefficients (r), together with the equations for the two regression-lines, are given in Table III. As an illustration, the scatter-diagram and the regression-lines for the correlation between $|V|$ and ρ are given in Figure 2. They may be compared with the cor-

[1] *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, **48**, 154, 1918.

[2] *Lunds Observatorium, Meddelanden*, Serie II, No. 29, 1923.

relation data for V and $\cos \lambda$ as given in Table III and Figure 1. For the nebulae, the correlation between $|V|$ and ρ can be regarded as equivalent to the correlation between V and ρ, as nearly all the radial velocities are positive. The same holds for $|V'|$, which is nearly identical with V', as the K-term obviously must not be subtracted.

From Table III we see that the only correlation which can be accepted with any confidence is that between radial velocity and position in the sky (V and $\cos \lambda$). For the nebulae, the correlation

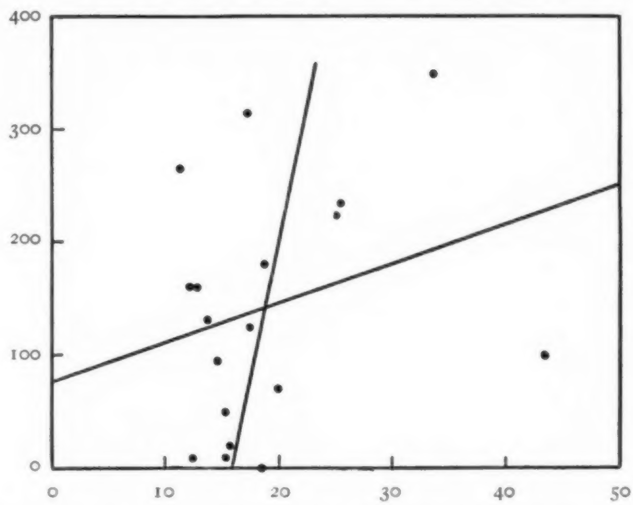

FIG. 2.—Scatter diagram showing correlation between radial velocities without regard to sign (ordinates) and distances in kiloparsecs (abscissae) for globular clusters.

coefficient is only $+0.54$, but the fact that the solar motion is nearly the same for the nebulae as for the clusters strengthens our belief that the correlation is real.

De Sitter's effect can be regarded as disproved by the clusters if their distances are of the same order as those of the nebulae. Silberstein's effect seems possible, but cannot be established by the data. It is significant, however, that the regression-line for the clusters, $|V| = 3.281\,\rho + 77$, does not go through the origin as expected from the theory. Silberstein's correlation is slightly improved if the radial velocities are corrected for the sun's motion.[1]

[1] This inclusion of the Magellanic clouds among the clusters does not materially alter the size of the correlation coefficient.

In conclusion we may say that we have found no sufficient reason to believe that there exists any dependence of radial motion upon distance from the sun. The only dependence fairly well established is one that is a function of position in the sky. This indicates a solar motion of 300 or 400 km/sec. in the direction $\alpha = 315°$, $\delta = +62°$, which agrees with the solar motion as determined from the asymmetrical velocity-distribution of stars in our neighborhood.

Mount Wilson Observatory
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THE ASYMMETRY IN STELLAR MOTIONS AS DETERMINED FROM RADIAL VELOCITIES[1]

By GUSTAF STRÖMBERG

ABSTRACT

Group-motion and velocity-dispersion for different classes of objects.—From all available radial velocities the group-motion and velocity-dispersion have been determined for all classes of objects with the aid of ellipsoidal distribution-functions. For spectral classes B0–M the stars have been grouped according to the quantity $H=m+5\log\mu$. The constants of the velocity-ellipsoids for fifty groups are given in Table I. The results show a continuous change in group-motion with internal velocity-dispersion, which is expressed by a quadratic relation between group-motion along an axis in the direction $\alpha=309\overset{\circ}{.}8$, $\delta=+57\overset{\circ}{.}1$, and the dispersion along the same axis. The group-motion projected on this axis varies regularly from 9.0 km/sec. for the Cepheids of long period to 300 km/sec. for fast-moving objects, globular clusters, and spiral nebulae.

Velocity-distribution for all cosmical objects.—The velocity-distribution for the whole collection of objects studied can be expressed as a product of two symmetrical distribution-functions, S_1 and S_2, with different centers of symmetry. The center of the distribution S_1, which itself is expressed as a sum of concentric, ellipsoidal distributions, corresponds to a velocity of 14.8 km/sec. in the direction $\alpha=85\overset{\circ}{.}7$, $\delta=-22\overset{\circ}{.}0$, and the center of the second distribution, S_2, is a velocity of 300 km/sec. in the direction $\alpha=127°$, $\delta=-56°$. The opposite vectors are the velocities of the sun relative to these centers. The first distribution can be interpreted as the distribution-law of velocities within our local system of stars, and the second as a velocity-restriction in a universal world-frame in which the clusters and spirals are statistically at rest; but other interpretations are also possible.

1. It has been known for some time that the *general* distribution of stellar motions cannot be represented by a single ellipsoidal distribution (the ellipsoidal theory) or by two or three spherical distributions (the two-stream and the three-stream theories), or by any kind of symmetrical frequency-functions. The existence of an asymmetry has been pointed out by Charlier, Boss, Raymond and Wilson, Adams and Joy, Oort, and the present writer, and is easily seen from the fact that stars of high velocity (larger than 100 km/sec.) do not move with the same frequency in opposite directions, but all move toward the region in the sky limited by galactic longitudes 134° and 343°, a region including little more than a hemisphere. This phenomenon is perfectly general, so far as we know, no exception having yet been found. The asymmetry is indicated

[1] *Contributions from the Mount Wilson Observatory*, No. 293.

by an increase in group-motion in a certain direction with increase in the velocity-dispersion within the group. In *Contribution* No. 275[1] it was shown that, with the exception of the B stars, a quadratic relation between group-motion and dispersion represents the data fairly well. If this quadratic relation be accepted, it leads to the conception of a velocity-restriction in a universal world-frame in which the spiral nebulae and globular clusters are statistically at rest. As it is of importance to find to what extent and how accurately the quadratic relation between group-motion and dispersion actually represents the known stellar velocities, an analysis has been made of all available radial velocities. The results of this analysis are given in the following pages.

2. Practically all radial velocities utilized in this study have been printed in the publications of the different observatories, except those of the Cepheids of short period and some of those of long period for which data have been furnished me by Adams and Joy. The radial velocities for the non-galactic nebulae and the globular clusters have been summarized in *Contribution* No. 292.[2] The method of procedure used was to find the group-motion, relative to the sun, of a certain class of object, and determine the velocity-distribution relative to the centroid of the group with the aid of an ellipsoidal frequency-function. In many cases it was possible to use physical criteria for the grouping of the stars, such as spectral characteristics, length of period of light-variation, and nebulous appearance, which enable us to form groups of objects of different average velocity. Absolute magnitudes have not been used, since we have insufficient knowledge of this characteristic of stars in the southern hemisphere. To obtain a grouping in classes of different velocity-dispersion, the apparent magnitudes and the proper motions were used in combination with the spectral types. In order to secure sufficiently large groups all stars within certain limits of spectral subdivision were taken together and subdivided according to values of the quantity $H = m + 5 \log \mu$, where m is the apparent magnitude on the Harvard scale, and μ the annual proper motion, taken in general

[1] *Astrophysical Journal*, **59**, 228, 1924.

[2] *Astrophysical Journal*, **61**, 353, 1925.

from Boss's *Catalogue* or *Cincinnati Publications* No. 18. It is easily seen that

$$H = M + 5 \log T - 8.378 \,, \tag{1}$$

where M is the absolute magnitude and T the linear tangential velocity in km/sec.

As we know that the average velocity increases in general with M, and as we have strong reason to believe that a large tangential velocity is correlated with a large radial motion, we may expect that the grouping according to H will enable us to form groups of the same spectral class but with different velocity-dispersions. When H has been used as the basis for grouping, we can regard the procedure as a dissection of a velocity-distribution in several overlapping component distributions, whereas, when we use the other criteria for the grouping, we obtain the constants for a group with certain physical properties. As the proper motions, except for the "moving groups," are not used in the computation of the constants of the velocity-distribution, the main effect of selection in the list of known radial velocities is to change the proportion of objects belonging to the different groups.

3. In the following we will denote the radial velocity of a star relative to the sun by V; the right ascension and declination by α and δ (1900.0); the galactic longitude and latitude by l and b; the equatorial velocity-components by ξ, η, ζ; and the galactic velocity-components by x, y, z. The galactic system used is the standard system, the north pole of the galactic plane having the co-ordinates $\alpha = 190°$, $\delta = +28°$. The equations for converting equatorial into galactic co-ordinates are thus:

$$\left.\begin{aligned} x &= +0.1736\,\xi - 0.9848\,\eta \\ y &= +0.4624\,\xi + 0.0815\,\eta + 0.8829\,\zeta \\ z &= -0.8695\,\xi - 0.1533\,\eta + 0.4695\,\zeta \end{aligned}\right\} \tag{2}$$

From the radial velocities of the stars in the group we determine first the group-motion, which has the components $\bar{\xi}$, $\bar{\eta}$, and $\bar{\zeta}$, from the equations of condition

$$\bar{\xi} \cos \alpha \cos \delta + \bar{\eta} \sin \alpha \cos \delta + \bar{\zeta} \sin \delta + K = V \,, \tag{3}$$

where K is Campbell's K-term. Changing the signs of $\bar{\xi}$, $\bar{\eta}$, and $\bar{\zeta}$, we obtain the components of the sun's motion relative to the objects studied. Denoting the right ascension and declination of the sun's apex by A_0 and D_0 and its velocity by v_0, we have

$$\left.\begin{aligned} v_0 \cos A_0 \cos D_0 &= -\bar{\xi} \\ v_0 \sin A_0 \cos D_0 &= -\bar{\eta} \\ v_0 \sin D_0 &= -\bar{\zeta} \end{aligned}\right\} \tag{4}$$

In the determination of the axes of the velocity-ellipsoids the directions of the principal axes have been assumed to be the same for all the groups. This is probably not strictly correct, but facilitates the computation and is often necessary on account of the small number of objects in several of the groups. The assumed directions of the principal axes are:

	α	δ	l	b
a	271°5	−17°1	341°1	− 0°7
b	327.6	+61.2	71.0	+ 5.6
c	188.9	+22.5	259.5	+84.4

The direction of the axis denoted by b is made identical with the axis of asymmetry as determined in *Contribution* No. 275, which is in fair agreement with one of the principal axes of the velocity-ellipsoids. The letters a, b, and c represent, in the following, the dispersions (square root of the mean square of the velocity-components) along the three axes, and determine, at the same time, the axes of the velocity-ellipsoids.

To determine a, b, and c from the radial velocities, the following method was used. On a map circles were drawn around the six points where the three axes intersect the sphere. These circles had in general a radius of 45°, although in some cases, when the number of stars was small, the radius was increased to 50° or 55°. The residuals from the equations of condition (3) were computed and the arithmetic means of these residuals, disregarding their signs, were formed for the stars within the circles just mentioned, opposite areas being combined. Three values, θ_1, θ_2, and θ_3, of the average residual velocity were thus found. From these values the dispersions a, b, and c were derived by making the assumption that the velocity-

distribution within each group could be represented by an ellipsoidal distribution-function. Such a function can be expressed by the formula

$$F(xyz)dx\,dy\,dz=\frac{N\,dx\,dy\,dz}{(2\pi)^{\frac{3}{2}}\,abc}\exp\left[-\frac{(x-\bar{x})^2}{2a^2}-\frac{(y-\bar{y})^2}{2b^2}-\frac{(z-\bar{z})^2}{2c^2}\right]. \quad (5)$$

The number of stars is here denoted by N; x, y, and z are the velocity-components along the principal axes, and the components of group-motion are denoted by $\bar{x}$, $\bar{y}$, and $\bar{z}$. From equation (5) we can derive the frequency-function of radial velocities for stars in $d\omega$, a small region of the sky, whose direction-cosines referred to the principal axes are γ_1, γ_2, and γ_3. We find, if we denote the residual (peculiar) radial velocities by V',

$$F(V')d\omega\,dV'=\frac{N\,d\omega\,dV'}{\sqrt{2\pi(a^2\gamma_1^2+b^2\gamma_2^2+c^2\gamma_3^2)}}\exp\left[\frac{-V'^2}{2(a^2\gamma_1^2+b^2\gamma_2^2+c^2\gamma_3^2)}\right]. \quad (6)$$

From equation (6) we derive

$$\overline{V'^2}=\sigma^2=a^2\gamma_1^2+b^2\gamma_2^2+c^2\gamma_3^2\,, \quad (7)$$

where σ is the mean square radial velocity for a small region in the sky, defined by its direction-cosines γ_1, γ_2, and γ_3.

If we form the mean square velocity for stars in circles of angular radius p, whose centers are at the intersections of the axes a, b, and c with the sphere, and denote by a'^2, b'^2, and c'^2 the means of the squares of the velocities for the three areas, we obtain, after combining opposite areas,

$$a'^2=a^2\,P+\frac{b^2+c^2}{2}\,(1-P)$$

$$b'^2=b^2\,P+\frac{a^2+c^2}{2}\,(1-P)$$

$$c'^2=c^2\,P+\frac{a^2+b^2}{2}\,(1-P)$$

$$P=\frac{1+\cos p+\cos^2 p}{3}$$

Solving these equations for a, b, and c, we find

$$\left.\begin{aligned}(3P-1)a^2&=(1+P)a'^2-(1-P)(b'^2+c'^2)\\(3P-1)b^2&=(1+P)b'^2-(1-P)(a'^2+c'^2)\\(3P-1)c^2&=(1+P)c'^2-(1-P)(a'^2+b'^2)\end{aligned}\right\} \quad (8)$$

For the particular case of $p=45°$ we have

$$a^2=1.4379\ a'^2-0.2189\ (b'^2+c'^2),$$

and similarly for b and c.

The values of a', b', and c' have been derived from the average velocities θ_1, θ_2, and θ_3 by multiplication with $\sqrt{\pi/2}=1.2533$. The effect of isolated large velocities is thus less than if the squares of the residual velocities had been used.

4. The detailed results of this analysis for fifty groups are given in Table I in the form of constants for the velocity-ellipsoids. This includes practically all known radial velocities except a few R and N stars, which are too few in number to be used as separate groups. The first twenty-nine groups include spectral types M, K, G, F, A, and B, and are all independent of one another, except group 27, which is a combination of groups 23–26. Groups 30 and 31 are the Cepheids of long and short period; group 32 contains the O stars, the elements being derived from Plaskett's radial velocities. The results from R. E. Wilson's study[1] of the space velocities of the O stars are given as group 33. Group 34 including stars with c characteristics (sharp spectral lines), is practically identical with the Pseudo-Cepheids; groups 35 and 36 are the bright-line nebulae (spectral class P). In the latter solution six objects of very high radial velocity have been omitted, the radial velocities determined by Campbell and Moore having been used.[2] Group 37 is based upon the radial velocities of the long-period variables, taken from Merrill's[3] determinations, and 38 is a group of Me stars with periods between 150 and 210 days, which in general have higher velocities than Me stars of other lengths of period.[4] Group 39 consists of the globular clusters, the solution given in *Contribution* No. 292,[5] based upon Slipher's determinations, having been used. Group 40 contains the spiral, elliptical, and irregular nebulae (by Hubble called "non-galactic nebulae") with continuous spectra, which are later referred to as "spirals." The

[1] *Astronomical Journal*, p. 841, 1924.

[2] *Lick Observatory Publications*, **13**, Part IV, 1918.

[3] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 215, 1923.

[4] Cf. Merrill, *op. cit.*, p. 44

[5] *Astrophysical Journal*, **61**, 353, 1925.

TABLE I

Group No.	1	2	3	4	5	6	7
Type	Mo–M9	Mo–M9	K4–K9	K4–K9	K4–M9	G9–K3	G9–K3
H	≤ -2.0	-1.9 to $+3.0$	≤ -2.0	-1.9 to $+3.0$	$\geq +3.1$	≤ -3.0	-2.9 to -2.0
No.	75	97	77	90	79	124	120
H	-3.64	-0.66	-3.68	-0.33	$+7.70$	-4.49	-2.45
$\bar{m}$	5.09	5.30	5.04	5.51	8.12	4.88	5.12
$\underline{\mu}$	0.0179	0.0643	0.0180	0.0679	0.824	0.0134	0.0306
$\bar{\xi}$	$+0.3$	-7.7	-8.4	$+4.2$	-10.0	-2.6	$+0.3$
$\bar{\eta}$	$+11.8$	$+13.6$	$+19.9$	$+29.3$	$+16.4$	$+17.0$	$+12.2$
$\bar{\zeta}$	-8.7	-19.3	-7.4	-17.5	-4.0	-10.4	-8.2
K	-0.3	$+0.8$	$+0.6$	$+4.6$	....	$+1.9$	$+2.3$
v_0	14.6	24.8	22.8	34.4	19.6	20.1	14.8
A_0	269°	300°	293°	262°	301°	270°	269°
D_0	+36°	+51°	+19°	+31°	+12°	+31°	+34°
v'_0	14.6	21.4	20.9	34.1	16.2	20.0	14.7
θ'	14.0	20.8	14.0	22.9	30.1	12.0	12.9
θ	14.3	19.9	14.0	21.9	29.6	11.8	13.2
x	-11.6	-14.7	21.0	-28.1	-17.9	-17.2	-12.0
y	-6.6	-19.4	-8.8	-11.1	-6.9	-9.1	-6.1
z	-6.1	-4.4	$+0.8$	-16.4	$+4.3$	-5.2	-6.0
a'	18.8	34.7	19.4	28.0	46.7	15.8	18.5
b'	17.4	23.1	15.2	24.5	24.8	13.8	14.9
c'	17.0	20.5	18.7	23.8	35.8	13.0	16.7
a	19.4	39.0	20.4	29.6	52.2	16.7	19.6
b	17.1	20.4	13.1	23.9	11.1	13.5	13.6
c	16.5	15.0	19.2	22.6	35.1	12.2	16.6
x'	-7.0	-3.6	-14.3	-19.4	-12.5	-10.8	-7.6
y'	-12.1	-24.6	-17.4	-25.5	-13.7	-16.7	-11.9
z'	-4.3	-0.6	$+3.5$	-12.5	$+6.5$	-2.6	-4.2

Group No.	8	9	10	11	12
Type	G9–K3	G9–K3	G9–K3	G9–K3	G9–K3
H	-1.9 to -1.0	-0.9 to 0.0	$+0.1$ to $+1.0$	$+1.1$ to $+3.0$	$+3.1$ to $+12.0$
No.	102	139	109	100	104
H	-1.40	-0.47	$+0.49$	$+1.87$	$+5.96$
$\bar{m}$	5.20	5.23	5.29	5.56	7.07
$\underline{\mu}$	0.0479	0.0724	0.1096	0.183	0.600
$\bar{\xi}$	-4.0	-1.4	$+1.6$	-9.4	-0.9
$\bar{\eta}$	$+12.4$	$+17.4$	$+15.7$	$+36.8$	$+15.0$
$\bar{\zeta}$	-7.1	-15.7	-14.5	-19.7	-21.4
K	$+0.6$	-1.2	$+2.9$	$+3.7$	$+3.2$
v_0	14.9	23.5	21.4	42.8	+26.1
A_0	288°	275°	264°	284°	273°
D_0	+28°	+42°	+43°	+27°	+55°
v'_0	14.3	23.0	20.8	41.7	23.6
θ'	17.0	18.4	23.3	24.6	23.3
θ	17.3	18.3	19.4	26.1	23.2
x	-12.9	-17.4	-15.2	-37.9	-14.9
y	-7.1	-13.1	-10.8	-18.7	-18.1
z	-1.7	-8.8	-10.6	-6.8	-11.5
a'	22.2	24.6	26.8[1]	38.9	32.2
b'	21.5	22.5	21.6	30.5	32.5
c'	18.2	18.9	20.9	31.8	23.5
a	23.2	26.0	28.9	41.9	33.7
b	22.0	22.7	20.4	28.0	34.3
c	16.3	16.5	19.2	30.4	18.3
x'	-7.9	-9.0	-8.2	-24.3	-4.5
y'	-12.5	-20.9	-18.2	-35.2	-24.5
z'	$+0.2$	-5.6	-7.9	-1.2	-7.8

TABLE I—*Continued*

Group No.	13	14	15	16	17	18
Type	G0–G8	G0–G8	G0–G8	G0–G8	F0–F9	F0–F9
H	≤ -2.0	-1.9 to $+1.0$	$+1.1$ to $+5.0$	$+5.1$ to $+10.0$	≤ -2.0	-1.9 to 0.0
No.	144	131	154	111	86	70
$\bar{H}$	-3.37	-0.53	$+3.19$	$+6.51$	-4.35	-0.74
$\bar{m}$	5.09	5.67	6.01	7.23	4.80	5.52
$\underline{\mu}$	0.0169	0.0575	0.2729	0.718	0.0148	0.0560
$\bar{\xi}$	$+4.0$	-0.3	-6.1	-11.9	$+5.2$	-1.8
$\bar{\eta}$	$+11.6$	$+17.2$	$+25.9$	$+37.4$	$+18.2$	$+14.7$
$\bar{\zeta}$	-11.8	-11.4	-8.1	-13.7	-11.5	-12.4
K	$+2.0$	$+1.6$	....	....	-0.1	$+0.5$
v_0	17.0	20.6	27.8	41.6	22.1	19.3
A_0	251°	271°	283°	287°	254°	277°
D_0	+44°	+34°	+17°	+19°	+31°	+40°
v_0'	15.9	20.6	26.5	39.2	21.5	18.9
θ'	12.1	15.3	23.2	32.9	11.2	14.9
θ	11.5	15.0	21.9	30.4	11.1	15.4
x	-10.7	-16.9	-26.5	-38.9	-17.0	-14.8
y	-7.6	-8.8	-7.8	-14.6	-6.2	-10.6
z	-10.8	-7.8	-2.5	-1.8	-12.7	-6.5
a'	16.5	23.2[2]	38.8	42.9	18.1	22.2
b'	13.3	17.3	25.0	34.5	11.9	18.8
c'	14.1	13.8	20.9	34.4	15.[3]	15.8
a	17.6	25.8	44.0	46.2	19.7	24.0
b	12.4	16.5	21.7	32.4	9.0	18.6
c	13.6	9.4	12.7	32.1	15.	13.2
x'	-5.8	-10.7	-19.6	-27.2	-12.0	-7.9
y'	-13.4	-16.9	-19.7	-31.2	-15.4	-17.2
z'	-8.8	-5.2	$+0.6$	$+3.1$	-10.4	-3.9

Group No.	19	20	21	22	23	24	25
Type	F0–F9	F0–F9[4]	F0–F9[6]	B6–A9	B6–A9	B6–A9	B6–A9
H	$+0.1$ to $+3.0$	$+3.1$ to $+5.0$	$+5.1$ to $+14.0$	≤ -3.0	-2.9 to -2.0	-1.9 to -1.0	-0.9 to 0.0
No.	246	82	38	188	138	133	130
$\bar{H}$	$+1.45$	$+3.84$	$+7.03$	-4.42	-2.43	-1.46	-0.44
$\bar{m}$	5.53	6.09	7.61	5.15	5.15	5.14	5.12
$\underline{\mu}$	0.153	0.355	0.766	0.0122	0.0305	0.0479	0.0773
$\bar{\xi}$	$+3.3$	-4.5	-36.6	-1.2	$+4.6$	$+3.2$	$+0.5$
$\bar{\eta}$	$+17.0$	$+35.8$	$+20.9$	$+13.4$	$+12.5$	$+16.5$	$+22.8$
$\bar{\zeta}$	-6.8	-0.3	-99.5	-17.4	-6.4	-3.0	-3.9
K	0.0	....	....	-0.3	-0.3	0.0	0.0
v_0	18.6	36.1	108.1	22.0	14.8	17.1	23.1
A_0	259°	277°	330°	275°	250°	259°	269°
D_0	+21°	0°	+67°	+52°	+26°	+10°	+10°
v_0'	18.1	31.2	67.9	20.3	14.0	15.8	21.6
θ'	16.2	25.0	71.1	10.1	10.2	12.7	10.9
θ	15.8	21.0	70.6	9.9	10.0	12.7	10.5
x	-16.2	-36.1	-27.0	-13.4	-11.5	-15.7	-22.3
y	-3.1	$+0.6$	-103.1	-14.8	-2.5	$+0.1$	-1.4
z	-8.6	-1.7	-18.1	-9.2	-8.9	-6.7	-5.7
a'	26.1	41.1[5]	110[5]	13.0	12.8	17.0	14.8
b'	20.0	23.8	81	12.4	13.8	16.6	12.0
c'	15.6	25.0	70	10.6	10.0	10.9	12.1
a	28.9	48.4	122.	13.0[7]	12.5[7]	17.7[7]	15.4[7]
b	19.3	14.8	76.	12.0	14.1	17.0	10.6
c	10.7	18.1	55.	8.7	7.2	5.7	10.7
x'	-12.7	-31.9	$+25.5$	-4.7	-8.9	-13.9	-19.0
y'	-11.7	-16.8	-105.0	-20.6	-9.0	-8.3	-12.6
z'	-6.9	$+0.9$	-1.6	-6.0	-7.6	-5.5	-3.8

TABLE I—*Continued*

Group No.	26	27	28	29	30	31	32
Type	B6–A9	B6–A9	B0–B5	B0–B5	Cepheids Per. > 2.0 days	Cepheids Per. < 0.7 days	O5–O9[18]
H	+0.1 to + 2.7	−2.9 to +2.7	≦ −4.0	−3.9 to + 0.1	....	....	....
No.	91	492	123	126	37[13]	26[16]	49
H	+ 0.89	− 1.03	− 5.55	− 2.75	....	....	....
$\bar{m}$	5.44	5.19	4.19	4.77	....	....	....
μ	0.123	0.0570	0.0113	0.0313	....	....	....
$\bar{\xi}$	− 5.2	+ 0.9	− 5.5	− 4.0	− 2.4	−44	− 2.0
$\bar{\eta}$	+25.5	+17.7	+17.6	+15.3	+10.3	+60	+21.4
$\bar{\zeta}$	− 6.2	− 4.6	−11.0	−13.8	− 4.6	−80	−28.4
K	+ 0.5	+ 0.2	+ 6.0[8]	+ 1.5[11]	− 1.7	....	+11.8
v_0	26.8	18.3	21.4	21.0	11.5	109	35.7
A_0	281°	267°	287°	285°	283°	306°	275°
D_0	+13°	+15°	+31°	+41°	+24°	+47°	+53°
v'	25.2	17.7	20.7	20.2	11.2[14]	92	32.8
θ'	13.4	12.1	7.3	9.7	11.4	57	22.2
θ	11.2	....	7.1	8.4	11.3	60	20.7
x	−26.0	−17.3	−18.3	−15.8	−10.5	−67	−21.5
y	− 5.8	− 2.2	−10.8	−12.8	− 4.3	−85	−24.3
z	− 2.2	− 5.7	− 3.1	− 5.4	− 1.7	− 8	−14.9
a'	12.8	....	8.7	9.3	15.8	73 [17]	22.0
b'	14.0	....	8.9	12.5	11.5	76	30.8
c'	11.6	....	8.9[9]	11.0[12]	....	88	....
a	12.1 [7]	16.4 [7]	8.1 [10]	7.4 [10]	17.3	74 [17]	19.6
b	14.1	14.6	8.4	13.0	10.0	74	33.9
c	10.0	9.1	8.4	10.6	(10)[15]	74	(19)[19]
x'	−20.1	−14.1	−10.9	− 7.7	− 7.2	−12	− 7.3
y'	−17.7	−10.9	−18.5	−19.4	− 9.0	−107	−33.5
z'	+ 0.5	− 4.0	− 0.2	− 2.3	− 0.2	+ 8	− 9.8

Group No.	33	34	35	36	37	38
Type	O5–O9[18]	c stars[21]	P	P	M1e–M6e	M2e–M5e P = 150–210 days
H	....	....	....	....	....	....
No.	41	66	107[24]	101[27]	86	13
H	....	− 5.77	....	....	....	....
$\bar{m}$	....	4.12	....	....	....	....
μ	....	0.0105	....	....	....	....
$\bar{\xi}$	+ 2.3	+ 3.2	+ 0.5	− 7.1	−14.1	− 45
$\bar{\eta}$	+20.2	+22.1	+18.8	+20.7	+47.2	+ 75
$\bar{\zeta}$	−22.0	−13.4	−21.3	−20.8	−42.7	−136
K	....	− 0.7	+ 2.0	+ 1.8	....	....
v_0	30.0	26.1	28.4	30.2	65.2	162
A_0	263°	262°	268°	288°	287°	301°
D_0	+47°	+31°	+49°	+44°	+41°	+ 57°
v'	28.5	25.9	27.0	28.3	62.2	133
θ'	....	12.1	40.0	28.8	45.5	80
θ	....	11.9	35.2	28.6	40.1	70
x	−19.5	−21.2	−18.4	−21.6	−48.9	− 82
y	−16.7	− 8.6	−17.0	−20.0	−40.4	−134
z	−15.5	−12.5	−13.3	− 6.8	−15.0	− 36
a'	....	13.5	56.2	41.2	52 [30]	97 [31]
b'	....	16.3	31.7	31.7	55	80
c'	....	11[22]	12[25]	12[28]	49	118
a	30 [20]	12.9	64.7	45.8	52	88 [32]
b	30	17.5	23.9	30.7	57	88
c	30	(12)[23]	(15)[26]	(15)[29]	46	88
x'	− 9.2	−14.6	− 9.1	− 9.5	−23.7	− 7
y'	−26.1	−19.4	−25.6	−28.6	−60.5	−161
z'	−11.5	− 9.6	− 9.4	− 2.3	− 5.6	− 11

TABLE I—*Continued*

Group No.	39	40	41	42	43	44
Type	Globular clusters	Non-galactic nebulae	Stars of maximum space-velocity[37]	Stars of radial velocity > 100 km	Calcium clouds	Ursa Major group
No.	18	44[35]	22	49	64	17
$\bar{\xi}$	− 79	−111	−118	− 80	− 1.6	+ 8.7
$\bar{\eta}$	+108	+160	+ 78	+147	+16.1	−11.3
$\bar{\zeta}$	−253	−284	−243	−166	−12.4	−12.0
K	+ 25	+622	...	...	...	...
v_0	286	344	281	236	20.4	18.6
A_0	306°	305°	303°	299°	276°	127°8
D_0	+ 62°	+ 56°	+ 60°	+ 45°	+37°	+40°0
v'_0	220	281	189	210	20.1	...
θ'	90	239	...	100	5.6	...
θ	96	252	...	101	5.1	...
x	−120	−177	− 97	−159	−16.2	+12.6
y	−251	−289	−263	−171	−10.4	− 7.4
z	− 67	− 61	− 23	− 31	− 6.8	−11.5
a'	123[33]	...	...	(173)	5.4	...
b'	131	...	...	(93)	7.1	...
c'	112	...	...	(60)	...	...
a	117[34]	300[36]	207[38]	...	5	...
b	117	300	108	...	5	...
c	117	300	82	...	...	...
x	+ 14	− 14	+ 40	− 58	− 9.2	+14.7
y	−285	−344	−278	−229	−17.7	− 2.3
z	− 22	− 8	+ 20	+ 5	− 4.2	−11.2

Group No.	45	46	47	48	49	50
Type	Taurus group	Praesepe group	Pleiades group	Perseus group	Scorpio-Centaurus group	Orion group
No.	39	10	11	29	131	17
$\bar{\xi}$	− 1.4	− 5.3	+ 1.3	− 6.3	− 1.9	+ 3.5
$\bar{\eta}$	+45.2	+40.7	+14.1	+17.0	+13.3	+17.0
$\bar{\zeta}$	+ 5.5	− 0.2	−13.4	− 9.9	−12.8	− 5.6
v_0	+45.6	41.0	−19.5	20.7	18.6	18.3
A_0	271°8	277°4	264°8	290°3	278°2	258°4
D_0	− 6°9	+ 0°3	+43°3	+28°7	+43°6	+17°9
x	−44.8	−41.0	−13.7	−17.9	−13.5	−16.2
y	+ 7.9	+ 0.7	−10.1	−10.2	−11.1	− 2.0
z	− 3.1	− 1.7	− 9.6	− 1.8	− 6.4	− 8.3
x'	−43.1	−36.4	− 7.2	−10.8	− 6.5	−13.3
y'	−14.7	−19.0	−16.7	−17.6	−17.0	−10.6
z'	− 0.8	+ 1.3	− 7.1	+ 1.0	− 3.8	− 6.7

NOTES

1. Group 10, one star with $V = +181$ km/sec. omitted.
2. Group 14, one star with $V = +183$ km/sec. omitted.
3. Group 17, from five stars only.
4. Group 20, four A stars included.
5. Groups 20, 21, $p = 50°$.
6. Group 21, four A stars and three G stars included.
7. Groups 22, 23, 24, 25, 26, 27, dispersions are corrected for the systematic effect of a mean error in V equal to ± 4.0 km/sec.
8. Group 28, probable error ± 1.13 km/sec.
9. Group 28, from ten stars only.

10. Groups 28, 29, corrected for systematic effect of a mean error $= \pm 3$ km/sec.
11. Group 29, probable error ± 0.74 km/sec.
12. Group 29, from five stars only.
13. Group 30, *w* Virginis, $V = -60$ km/sec. omitted.
14. Group 30, a direct solution assuming the apex to have the value $A_0 = 270°$, $D_0 = +30°$ gives $V_0 = 12.3 \pm 1.56$, $K = -1.8 \pm 1.81$.
15. Group 30, assumed value.
16. Group 31, VX Herculis, $V = -380$ km/sec. omitted.
17. Group 31, $p = 50°$, spherical distribution assumed. These values are corrected for the effect of a mean error in V of ± 15 km/sec.
18. Groups 32, 33. Plaskett's classification.
19. Group 32, assumed value.
20. Group 33, spherical distribution assumed by Wilson.
21. Group 34, this group includes Pseudo-Cepheids.
22. Group 34, from three stars only.
23. Group 34, assumed value.
24. Group 35, no omissions except Magellanic clouds.
25. Group 35, from three objects only.
26. Group 35, assumed value.
27. Group 36, six nebulae of velocities higher than 100 km/sec. omitted.
28. Group 36, from three objects only.
29. Group 36, assumed value.
30. Group 37, $p = 50°$.
31. Group 38, $p = 55°$.
32. Group 38, spherical distribution assumed.
33. Group 39, $p = 50°$.
34. Group 39, corrected for a mean error in V of ± 25 km/sec. Spherical distribution assumed.
35. Group 40, solution by groups. Magellanic clouds included without *K*-term.
36. Group 40, spherical distribution assumed. These values are subject to great uncertainties, particularly as the *K*-term is probably a variable quantity with large dispersion.
37. Group 41, determination based upon space velocities.
38. Group 41, the directions of the three principal axes are in order $L_1 = 164°$, $B_1 = +6°$; $L_2 = 73°$, $B_2 = +8°$; $L_3 = 291°$, $B_3 = +80°$.

constants given are those derived in the last-mentioned paper by a solution in groups, omitting N.G.C. 584 but including the Magellanic clouds without K-term. Group 41 is composed of stars, mostly of spectral type F, with large space velocities, which seem to form a separate group. All stars with radial velocities larger than 100 km/sec. are taken together as group 42. Since the selection of the last two groups is based directly upon the velocities of the stars in the group, they are not comparable with the other groups, but the group-motion may still be of some value. Group 43 contains the stars with stationary calcium-lines, which, according to Hartmann[1] and Plaskett,[2] are due to calcium clouds in space. This system of clouds seems to form a group with very small velocity-dispersion and can, so far as its motion is concerned, be regarded as equivalent to the other moving groups, the elements of which are given in columns 44–50. The constants for the Ursa Major group are means of Bottlinger's and Rasmuson's determinations; the elements for the Pleiades, Perseus, Scorpio-Centaurus, and Orion groups are taken from Rasmuson's study;[3] for the Taurus group, Boss's elements are given.

[1] *Astrophysical Journal*, **19**, 268, 1904.

[2] *Publications of the Dominion Astrophysical Observatory*, **2**, 287, 1924.

[3] *Lunds Observatorium, Meddelanden*, Serie II, No. 26, 1921.

For the Praesepe group, the constants are based upon a parallax of 0".007 and a radial velocity of +33 km/sec., determined by Adams and Joy.

The first line in Table I gives the current number of the group. This is followed by the types of objects studied, the limits of $H = m + 5 \log \mu$ (for those cases in which H has been used as a basis of grouping), and the number of objects in the group. Then follow the mean H, the mean apparent magnitude, and the geometric mean of the proper motions. This last quantity μ is defined by the equation

$$\mu = 10^{0.2(\bar{H} - \bar{m})} = 10^{\overline{\log \mu}} .$$

The equatorial components of the group-motion $(\bar{\xi}, \bar{\eta}, \bar{\zeta})$ and the K-term determined from the equations of condition (3) are next given. In several cases, especially for faint stars of large proper motion (large H), the observed stars are mainly in the northern hemisphere and the K-term is thus difficult of determination and is supposed to be small or zero. The sun's velocity relative to the group is next given in polar equatorial co-ordinates (v_0, A_0, D_0). The sun's velocity projected on an axis in the direction of the standard apex $(A_0 = 270°, D_0 = +30°)$ is denoted by v_0'. This may be used in computing mean parallaxes, but the mode of selection of the stars must be taken into account. θ' is defined by the equation

$$\theta' = \sqrt{2/\pi}\, \sigma = 0.7979\, \sigma ,$$

σ^2 being the mean of the squares of the residual velocities. It can be compared with θ given in the next line, which is the average residual radial velocity. These two values would be identical if the distribution could be exactly represented by an ellipsoidal distribution-function.

The velocity-components of the group-motion referred to the standard galaxy are denoted by x, y, and z. The mean square velocity in the areas of radius p around the principal vertices are denoted by a', b', and c'; a, b, and c are the dispersions along the three principal axes, and are identical with the axes of the velocity-ellipsoids. The last three lines show the components of the group-motion projected on three new axes (x', y', z'), the directions of which will be given later.

On account of the nature of the grouping, very few omissions had to be made, and these are all indicated in the notes.

5. We see immediately that the dispersion, defined by a, b, c, or by θ, increases in general with the value of H, which shows, as had been anticipated, that the velocity-components are not independent of one another. Part of this increase is due to the correlation between the absolute magnitude, which enters into H (cf. eq. [1]), and the velocity-dispersion. We see further that in general v_0 increases with H and with the dispersion, a correlation which we should expect if the distribution were asymmetrical.

The small value for the sun's velocity relative to Cepheids of long period ($v_0 = 12.3$ km/sec. Cf. note 14) is of special importance, as this affects the mean parallax of these stars and consequently the distances of the clusters, for the reason that the absolute magnitudes of the Cepheids in the clusters are assumed by Shapley to be the same as those of the Cepheids of the same period in our local system. It also affects Hubble's estimated distances of the spirals, which are based upon the same assumption. A change of solar motion from 20.0 to 12.3 km/sec. produces a change in the absolute magnitude of $+1.06$ and decreases the distances by 38 per cent.

The change in the declination of the sun's apex with increasing H and with increased dispersion is also of interest. In general, the declination increases with H, but this increase is counteracted by the abnormal group-motion for the dwarfs referred to later. It is possible that the observed variation in the declination of the sun's apex as determined from proper motions is almost entirely due to the asymmetrical velocity-distribution. Since apparently faint stars, which, of course, are not the same as dwarf stars, are in general intrinsically fainter than the apparently brighter stars, and presumably have a higher velocity-dispersion, we may expect an increase in the declination of the sun's apex with decreasing apparent brightness. On account of the grouping used here and the small number of faint stars with small proper motion, the rate of this shift with apparent magnitude cannot be determined.

The general result of this study is that the sun's motion is not a constant vector but changes greatly with the class of object to which the motion is referred. Our next purpose is to find if

any definite laws can be established for the variation in group-motion. Figure 1 is a diagram giving the projections of the velocity-ellipsoids in the plane of the standard galaxy (xy-plane). The sun is at the origin, and a line from the origin to the center of an ellipse or circle indicates the group-motion projected on the galactic plane,

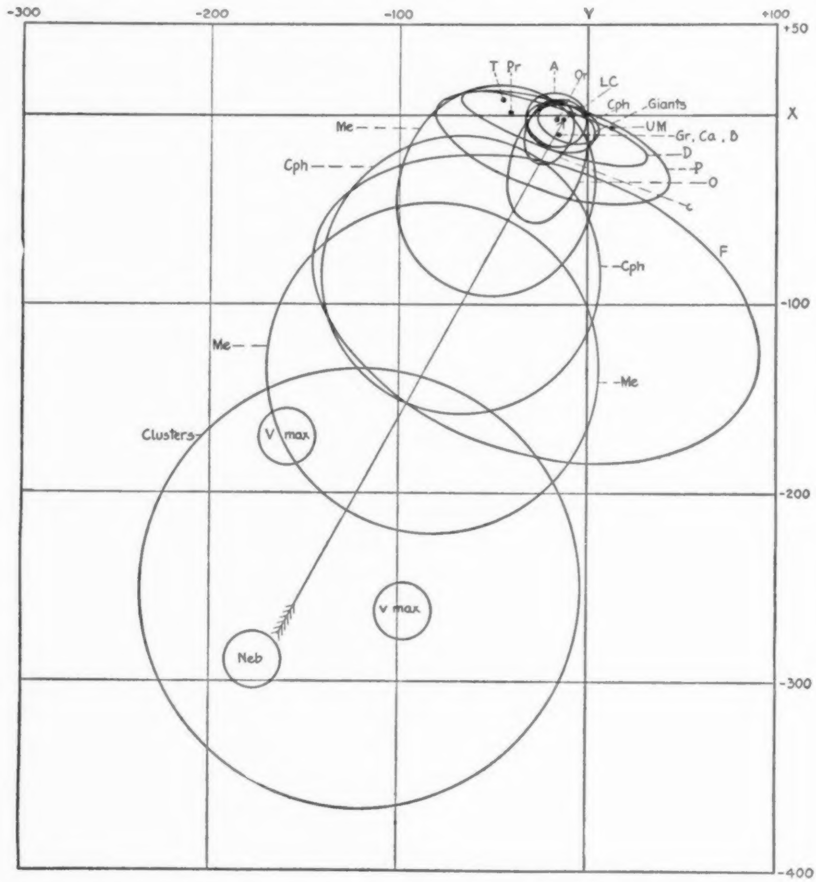

FIG. 1.—Projection of a number of velocity-ellipsoids on the galactic plane. The sun is at the origin, and a vector from the origin to the center of an ellipse or circle indicates the velocity of a certain group of objects, projected on the galactic plane. The principal axes of the ellipses are equal to the velocity-dispersion in the direction of the axes, except for the circles marked "V max," "v max," and "Neb," which indicate simply the group-motion. The points refer to moving groups with small internal velocity-dispersions.

and the size of the axes of the ellipses indicates the dispersions a and b. To avoid confusion only a few samples from the first twenty-nine groups are included in the diagram. Stars of spectral types F0–M with $H < -2.0$ have been combined in one group, the constants for which are as follows:

No.	x	y	z	a	b	c	x'	y'	z'	
626	−14.5	−7.4	−7.0	18.7	13.0	15.1	−14.4	−9.2	−4.9	km/sec.

This group is marked "Giants" in the diagram. Further, the velocity-ellipsoids for groups 5, marked *D* (dwarfs), 21 (*F*), 27 (*A*), 30 (*Cph*), 31 (*Cph*), 32 (*O*), 34 (*c*), 35 (*P*), 37 (*Me*), 38 (*Me*), and 39 (Clusters) are also shown in the figure. The group-motions for groups 40, 41, and 42 are marked by small circles with the notation *Neb*, v_{max}, and V_{max}, respectively. The velocity-vectors for the Ursa Major, Taurus, Praesepe, and Orion groups are indicated by points marked *UM*, *T*, *Pr*, and *Or*, respectively. The velocities of the Calcium Clouds, the Pleiades, Perseus, and Scorpio-Centaurus groups, and the B stars (groups 28 and 29) are very nearly alike, and have been combined into one point marked *Gr*, *Ca*, *B*, which has the co-ordinates $x = -15.9$, $y = -10.9$, $z = -5.5$ km/sec.

From the diagram we see that the centers of the velocity-ellipses show a marked shift toward the third quadrant of the xy-plane as the ellipses grow larger. This is in accordance with the results from space velocities and radial motions previously given in *Contribution* No. 275.[1] This shift determines a certain axis whose galactic longitude is now found to be 61°.5 with an uncertainty of about ±5°. Projecting the velocity-ellipsoids on a plane through a line in galactic longitude 61°.5, we find the galactic latitude to be +9°.0. This new projection is shown in Figure 2. The axis, of galactic longitude 61°.5 and latitude +9°.0, is the direction of motion of objects of small velocity-dispersion relative to objects of large dispersion, and in my former investigation was estimated to have the direction $l = 71°$, $b = +5°$. The projections of this vector in the xy-plane and in the plane through the z-axis are indicated by arrows in Figures 1 and 2.

We will now project the velocity-ellipsoids on three new axes

[1] *Astrophysical Journal*, **59**, 228, 1924.

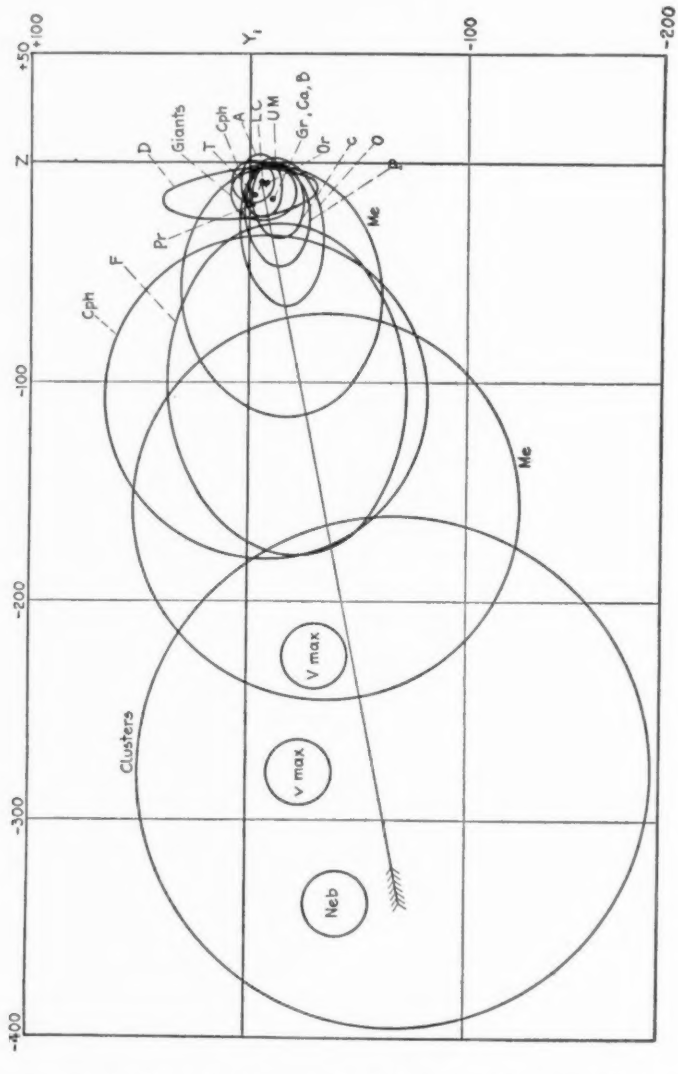

FIG. 2.—Same as Fig. 1, but the projection is here on a plane perpendicular to the galactic plane. The y'-axis lies in galactic longitude $61\overset{\circ}{.}5$, and the z-axis points toward the north pole of the galactic plane.

(x', y', z'), of which y' has the direction just determined and the x'-axis has zero galactic latitude. The directions of the new axes are:

	l	b	a	δ
x'	331°5	0°0	265°7	−24°9
y'	61.5	+ 9.0	309.8	+57.1
z'	241.5	+81.0	185.4	+20.0

The components of the group-motion projected on these axes appear in the last three lines of Table I. The axes coincide nearly with the directions of the principal axes assumed at the beginning.

If we study the velocity-components x', y', and z' we find that z' varies very little throughout the groups studied, the values with the largest deviations having the smallest weights. The x'-component varies somewhat more, but its variation does not seem to be systematic, except that it has an algebraically smaller value for the dwarf stars (groups 5, 11, 15, 16, 20) than for the giants. This may be due to the selection of stars of large proper motion in combination with a correlation between radial and transverse motion. The same applies to the variation of x' with H among the A stars (groups 23 to 26) where, in the absence of an asymmetry, the dispersion along the x'-axis produces a change in x' with H.

The y'-component varies enormously, from −9.0 for the Cepheids[1] of long period to −344 for the spirals. The variation in y' does not seem to be a function primarily of spectral type, absolute magnitude, or mass, but is correlated with the dispersion (b) along the same axis, and the result of the plotting is shown in Figure 3, where *all* the groups are indicated (except 23, 24, 25, and 26, which are combined in group 27). The relation between y' and b can be represented, throughout the whole range, by the parabola

$$\left.\begin{aligned} y' &= -pb^2+\beta' \\ p &= 0.0192 \text{ sec./km} \\ \beta' &= -10.0 \text{ km/sec.} \end{aligned}\right\} \qquad (9)$$

A slight ambiguity is involved here. It is, of course, possible to find a relation between y' and the dispersion along the two other

[1] Groups 23 and 24 (A stars) show numerically smaller values of y' than the Cepheids, but the combined group (27) has $y' = -10.9$.

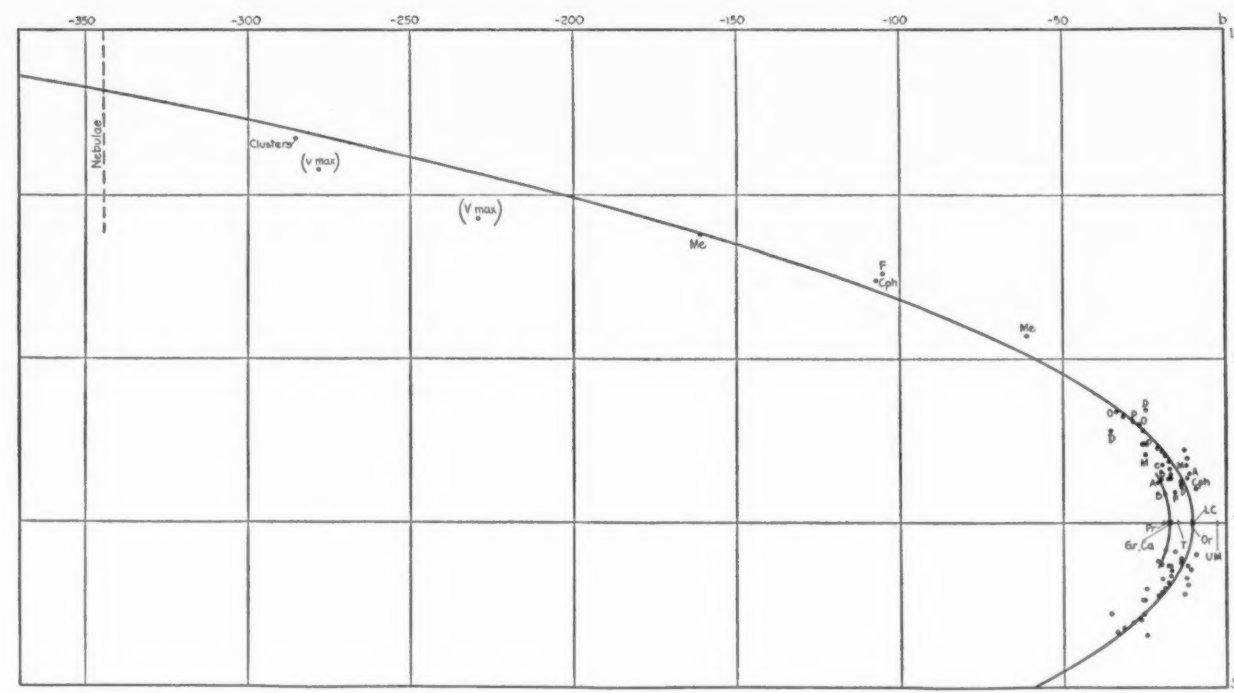

FIG. 3.—Relation between group-motion (abscissae), projected on an axis y' in galactic longitude $61^\circ.5$ and latitude $+9^\circ$, and the velocity dispersion parallel to this axis (ordinates). This relation is expressed by a parabola.

axes, a and c, but this does not lead to any relation that can be simply interpreted like the one just given. Further, in the cases where a and c differ considerably from b, as in groups 5 and 21, it is the dispersion b, and not a or c, which varies regularly with y'.

The case of the dwarfs of type K4 to M (group 5) is of special interest. Here we have a group of large velocity-dispersion (average velocity $\theta = 29.6$), which gives a small value of y' (-13.7 km). But the velocity-ellipsoid is very elongated, and the axis b is of the right size for the corresponding value of y'.

As found in the previous study, the B stars (groups 28, 29) fall off the parabola in Figure 3. The same applies to the A stars for $H < -3.0$ (group 22), which have a large group-motion and a fairly small dispersion. The sudden change in group-motion among the A stars for the value $H = -3.0$, i.e., in passing from group 22 to 23, is certainly real and very illuminating. It shows that the apparently bright A stars of very small proper motion, which of course are mainly of exceedingly high luminosity, share the motions of the B stars and probably are in some way connected with the B-star system. On the other hand, we see that the moving groups, Perseus, Pleiades, Scorpio-Centaurus, and the Calcium clouds have the same group-motion as the B stars; in fact a large proportion of the B stars is included in the Scorpio-Centaurus group. We can even draw a new parabola through the points represented by groups 22, 28, and 29, parallel to the principal parabola, which cuts the y'-axis at the point represented by the four moving groups just mentioned. The c stars are represented by a point intermediate between the two parabolas and may include stars belonging to the two systems. Although the existence of the second parabola cannot be proved, there is no doubt that the B stars, the most luminous A stars, and the four moving groups mentioned have approximately the same group-motion, and do not fit so well in the general relation between group-motion and dispersion as the other objects. The scattering of the points around the parabola may be due to a mixture of two systems of objects.

The point where the principal parabola cuts the y'-axis has the co-ordinate $y' = -10.0$ km/sec. The mean of the x'- and of the z'-co-ordinates in combination with this last value of y' constitutes the

apex of a parabola in a four-dimensional space (x', y', z', b) and gives us a fundamental velocity whose components we denote by α', β', and γ'. This point, which corresponds to the "limiting center" in my previous study, is marked *L.C.* in the three diagrams, and has the following co-ordinates:

$\alpha'=x'=-10.1\pm0.47$	$x=-13.3$	$\xi=+\ 1.0$	$\alpha=85^\circ.7$
$\beta'=y'=-10.0\pm1.0$	$y=-\ 3.3$	$\eta=+13.7$	$\delta=-22^\circ.0$
$\gamma'=z'=-\ 4.1\pm0.25$	$z=-\ 5.6$	$\zeta=-\ 5.5$	$v=14.8$

The velocity-vector corresponding to the limiting center is very nearly the same as the most frequent velocity found for later-type stars. The value for this velocity given in *Contribution* No. 245, page 28,[1] is $\alpha=86^\circ.2$, $\delta=-30^\circ.8$, $v=13.4$ km/sec., or $x'=-7.6$, $y'=-10.2$, $z'=-4.1$ km/sec.

The asymmetrical position in velocity-space of the most frequent velocity has also been pointed out by Boss, Raymond, and Wilson.[2] The small value for the sun's velocity when referred to the center of mass of a number of stars, as determined by Balanowsky and Samoïlova,[3] is due to the fact that in general the stars of high luminosity, which have been assigned the greater mass, have a smaller group-motion relative to the sun than those of less luminosity. The solar motion, whether we take the masses of the stars into account or not, is not a definite velocity-vector, but is greatly affected by the relative proportion of giant and dwarf stars in the system relative to which the sun's motion is determined. This phenomenon has also been found by Seares[4] by comparison of the secular parallaxes with the accepted luminosity and density functions.

The masses of the stars do not seem to enter in any simple way. Although, as has been pointed out by Halm and more thoroughly investigated by Seares,[5] there is a tendency to equipartition of kinetic

[1] *Astrophysical Journal*, **56**, 292, 1922.

[2] *Astronomical Journal*, **35**, 26, 1923.

[3] *Astronomische Nachrichten*, **222**, 290, 1924.

[4] *Mt. Wilson Contr.*, No. 281; *Astrophysical Journal*, **60**, 50, 1924.

[5] *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 165, 1922.

energy among several classes of stars, there are several prominent exceptions. The Cepheids of short period, although less luminous and perhaps of less mass than the Cepheids of longer period, can hardly have masses considerably below those of the dwarf stars. The same applies to the Me stars and to a number of F stars of large proper motion. The O stars and the c stars, for instance, which have very large masses, show a larger dispersion b and a larger group-motion than the dwarf stars of types K4–M9.

6. The result of this analysis is that we have found a constant vector whose components are α', β', and γ', and a relation between group-motion and dispersion defined by the equations

$$\overline{x'}=\alpha' , \qquad \overline{y'}=-pb^2+\beta' , \qquad \overline{z'}=\gamma' ,$$

which define an axis in space[1] and a numerical relationship. If we accept this relationship, we can now write down the frequency-function for the whole collection of objects in Table I, excluding the B-star system previously defined and the Ursa Major, Taurus, Praesepe, and Orion groups, which probably must be regarded as individual objects, in the following form:

$$F(x'y'z')dx'dy'dz'$$

$$=\frac{dx'dy'dz'}{(2\pi)^{\frac{3}{2}}}\sum_{\nu}\frac{N_\nu}{a_\nu b_\nu c_\nu}\exp\left[-\frac{(x'-\alpha')^2}{2a_\nu^2}-\frac{(y'-\beta'+pb_\nu^2)^2}{2b_\nu^2}-\frac{(z'-\gamma')^2}{2c_\nu^2}\right] . \quad (10)$$

This equation can be simplified, and we write

$$F=\exp\left[-p(y'-\beta')\right]\sum_{\nu}A_\nu\exp\left[-\frac{(x'-\alpha')^2}{2a_\nu^2}-\frac{(y'-\beta')^2}{2b_\nu^2}-\frac{(z'-\gamma')^2}{2c_\nu^2}\right] . \quad (11)$$

There is one fact, however, which we now wish to emphasize. Besides stars and nebulae which are all within about 1000 parsecs we have in our study included the globular clusters and the spirals (non-galactic nebulae). The clusters are certainly very distant objects and, according to Hubble and Lundmark, the spirals are at

[1] The quantity p can be regarded as a vector, the direction of which is toward the negative y'=axis.

even greater distances. There may still be some doubt about the distances of the spirals, especially in view of van Maanen's determinations of the internal motions, but their distances are at least above 1000 parsecs, and they may be at distances of the order of 300,000 parsecs (a million light-years). The group-motion and the dispersion for the clusters is approximately the same as that of the fastest-moving objects in our local system. The velocity-ellipsoid for the clusters seems to form the limit in the sequence of velocity-ellipsoids. Group 41 has even larger dispersion (a) along the x'-axis than the dispersion for the clusters, but this group as well as group 42 is selected on the basis of motion, and as the dispersion is in general largest along the x'-axis, we have made a selection of stars of large motion along the x'-axis. The spirals have about the same group-motion as the clusters, provided we omit the highest velocity (+1800 km/sec. for N.G.C. 584), but the dispersion for the spirals is much larger than for the clusters and the high velocity-stars, and consequently does not fit the parabola in Figure 3. It is probable that the K-term, which for the spirals has the enormous value of 600 km/sec., has a physical meaning and varies widely, being small for the Andromeda nebulae and the Magellanic clouds but very large for N.G.C. 584. If the dispersion in the K-term is very large, the actual dispersion in velocity for the spirals may be of the same order as that of the clusters.[1] As long as we do not know the cause of the large K-term or any law for its variability, we cannot correct for the extra dispersion. It does not seem to vary appreciably with distance, as would be the case if it were solely an effect of curvature of space-time.[2]

Equation (11) is a product of a symmetrical function and another function of the form $\exp[-p(y'-\beta')]$, which is found to be constant for all the groups. It expresses the law for the increase in dispersion with group-motion as we pass along the negative y'-axis, but it does not indicate whether there is a limit to the sequence of velocity-ellipsoids or not. The limiting velocity-ellipsoid is now

[1] A dispersion in the *K*-term of 275 km/sec. would bring the velocity-dispersion for the spirals down to 120 km/sec., which is about the same as that for the clusters.

[2] Cf. *Mt. Wilson Contr.*, No. 292; *Astrophysical Journal*, **61**, 353, 1925.

found to be equal to the velocity-ellipsoid for the clusters, and we can find an explicit expression for this limit by a simple transformation of equation (11).

We write

$$\left.\begin{aligned}
F &= S_1 S_2 \\
S_1 &= \sum_\nu B_\nu \exp\left[-\frac{(x'-\alpha')^2}{2}\left(\frac{1}{a_\nu^2}-\frac{1}{l_1^2}\right)-\frac{(y'-\beta')^2}{2}\left(\frac{1}{b_\nu^2}-\frac{1}{l_2^2}\right)\right. \\
&\qquad\qquad\qquad\qquad\left.-\frac{(z'-\gamma')^2}{2}\left(\frac{1}{c_\nu^2}-\frac{1}{l_3^2}\right)\right] \\
S_2 &= \exp\left[-\frac{(x'+x_0')^2}{2l_1^2}-\frac{(y'+y_0')^2}{2l_2^2}-\frac{(z'+z_0')^2}{2l_3^2}\right] \\
x_0' &= -\alpha', \qquad y_0' = -\beta'+pl_2^2, \qquad z_0' = -\gamma'
\end{aligned}\right\} \quad (12)$$

S_1 and S_2 both represent symmetrical distributions, the centers of symmetry of the first function being α', β', γ' and the center of the second function $-x_0'$, $-y_0'$, $-z_0'$. When the dispersions, represented by the quantities a, b, and c, increase up to their limits l_1, l_2, and l_3, the center of the distribution moves regularly from the center of the first distribution S_1 to that of the second distribution S_2. The name "limiting center" applied to α', β', γ' in my former study is due to this circumstance. The limits of the dispersion, l_1, l_2, and l_3, may well be equal, in which case the distribution S_2 is spherical, as seems to be the case for the clusters.

Putting the limiting value of the group-motion equal to that derived from clusters and spirals, we find this limit to be about 300 km/sec. in the direction of the negative y'-axis, and we have thus $y_0' = +300$ km/sec. If we use the known value of β' and p in combination with the dispersion for the cluster, we have $y_0' = pb^2 - \beta' = +273$. The group-motion for group 41 gives $y_0' = +278$. The velocity-vector x_0', y_0', z_0' is the velocity of the sun relative to the clusters, spirals, and fast-moving objects in general, and we can assign the following values to the components of this motion:

$$\left.\begin{aligned}
x_0' &= -\alpha' = +10.1 \text{ km/sec.} \\
y_0' &= +300 \pm 50 \quad \text{km/sec.} \\
z_0' &= -\gamma' = +4.1 \text{ km/sec.}
\end{aligned}\right\} \quad (13)$$

In polar co-ordinates we find the velocity of the sun relative to the center of the distribution S_2 to be 300 km/sec. in the direction

$$\left.\begin{array}{ll} \alpha=307^\circ & l=60^\circ \\ \delta=+56^\circ & b=+10^\circ \end{array}\right\} \qquad (14)$$

The uncertainty in this apex is about 5° in galactic longitude and 3° in galactic latitude.

When y_0' is known we can determine the value of l_2 from the equation $pl_2^2=y_0'+\beta'$. We have thus $l_2=123$ km/sec.

7. We will now attempt a physical interpretation of the two velocity-distributions S_1 and S_2. The function S_1 is a sum of concentric, ellipsoidal distributions, the co-ordinates of the center being α', β', γ', and can be regarded as a velocity-restriction relative to this center. This means simply that high velocities are less frequent than the smaller velocities, and that velocities of the same size are equally frequent in opposite directions, provided they are referred to this center. The same applies to the distribution S_2, which is another symmetrical distribution, its center having the co-ordinates $-x_0'$, $-y_0'$, $-z_0'$. The fundamental axis (y') is a line joining these two centers, and is represented by the arrows in Figures 1 and 2. Using the same terminology as for the distribution S_1, we may say that S_2 represents an additional velocity-restriction in a co-ordinate system in which the globular clusters and the spirals are statistically at rest. On account of its great size, and the apparent impossibility of ever observing a larger system, we will call this system the "world-frame." The distribution of velocities for almost all known cosmical objects can thus be represented by a product of two symmetrical distribution-functions. The first of these (S_1) represents probably the internal motions in our local system of stars, or rather what they would be if S_2 were not acting while S_2 represents a statistical restriction of velocities in a world-frame of enormous dimensions. This local system is probably identical with "Kapteyn's Universe," the dynamics of which has been studied by Kapteyn[1] and by Jeans.[2] The B stars studied and the A stars of high luminosity probably belong to Shapley's local star cloud, whose existence is indicated by

[1] *Mt. Wilson Contr.*, No. 230; *Astrophysical Journal*, **55**, 302, 1922.

[2] *Monthly Notices*, **82**, 122, 1922.

the fact that the plane of concentration for the bright B stars is inclined about 10° to the galactic plane.[1] The existence of the second distribution (S_2) indicates that the motions of the stars in the local (Kapteyn) system are determined not only by the gravitational field due to this system, but also by an outside field connected with the system of clusters and spirals.

The nature of the connection between the stars in the local system and the world-frame is not known, but there are several ways of explaining this connection.

The effect may be due to encounters between stars in the local system and stars belonging to the world-frame. This does not seem probable on account of the extremely low density of the larger system, and because, further, we should expect a strong correlation between group-motion and the masses of the stars. It can hardly be due to a system of dust at rest relative to the world-frame, as we should then expect a correlation between group-motion and the density of the stars.

The general gravitational field due to the larger system combined with the field of the local system may possibly produce a velocity-distribution similar to the one observed. On account of the low density and elongated shape of the larger system this explanation seems very improbable.

Another explanation is possible if we are allowed to regard the additional velocity-restriction as due to the action of a stationary medium in which the whole cosmical system is imbedded. The velocity vector (x'_0, y'_0, x'_0) is then the velocity of the sun relative to this medium, and l_1, l_2, and l_3 (which we will assume to be alike and equal to l) determine the effectiveness of the restrictive action, presumably dependent upon the mechanism of the interaction between the stars—or the electrons within the stars—and the medium. The dispersion in the spherical distribution S_2, whose center is at rest in the medium, is then $l = 123$ km/sec. and the most frequent velocity is $\sqrt{2}\, l = 174$ km/sec. The stars in our local system have in general a velocity of about 300 km/sec. relative to the world-frame, but they belong together and must in this connection be regarded as one

[1] *Astronomische Nachrichten*, Jubiläumsnummer, p. 25, 1921.

unit, just as we have regarded the globular clusters as individual objects.

A popular description of the velocity-distribution and a discussion of the possible interpretations are given in *Scientific Monthly*, November, 1924.

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Mount Wilson Observatory
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DISCOVERY AND OBSERVATIONS OF STARS OF CLASS Be[1]

By PAUL W. MERRILL, MILTON L. HUMASON, and CORA G. BURWELL

ABSTRACT

Stars of class B whose spectra contain emission lines of hydrogen.—Three reasons for studying these spectra are suggested: (1) possible assistance in the interpretation of the typical nova spectrum; (2) the bright lines appear to be sensitive indicators of conditions in stellar atmospheres; (3) unusual properties of hydrogen atoms may be involved.

The *discovery* of these stars by other observers has usually been made from the $H\beta$ line, but $H\alpha$ serves better because of its greater intensity. By photographing the red portion of the spectrum we have discovered 90 early-type stars with hydrogen emission. These are listed in Table I. Eighty-four of the stars were found from objective-prism observations with the 10-inch Cooke telescope on Mount Wilson.

Data concerning the Hα line in *Be stars*, as well as in certain *nebulae*, *Wolf-Rayet stars*, and *novae*, are contained in Tables II, III, and IV. The *ultra-violet hydrogen lines* in the Be stars are usually seen in absorption even when $H\alpha$ is bright. Estimates of their intensity are included in Table II.

The *blue-violet portion of the spectrum* has been observed with slit spectrographs attached to the large reflectors. Descriptions of $H\beta$, $H\gamma$, and $H\delta$, and various other data including *radial velocities* of a few stars, are given in Table V.

The *discussion* brings out the fact that our objective-prism spectrograms have yielded *more new bright-line stars than the number previously known within the areas observed.* It is suspected that in some of these objects the bright lines have come into existence since the Harvard spectroscopic surveys. A study of the *distribution* of Be stars shows a tendency for them to fall into groups near the center line of the Milky Way. Four of these groups occupy areas which are also rich in Wolf-Rayet stars. The *frequency of bright-line stars in the various spectral subdivisions* is examined and classes B0 to B5 are found to be strongly favored.

INTRODUCTION

The sun and nearly all the stars have spectra in which the chief features are absorption lines and bands. This type of spectrum may accordingly be considered the normal one. Less than 0.5 per cent of all the stars whose spectra have been examined are known to possess well-marked emission lines, and these objects have long been regarded as stellar curiosities. It is an interesting fact that most of the bright-line spectra are associated with either the hottest or the coolest stars. More than 95 per cent of the known bright-line stars fall in classes B and O on the one hand, or in classes M, S, and N on the other. Among classes A, F, G, and K, bright-line stars, while

[1] *Contributions from the Mount Wilson Observatory*, No. 294.

not entirely unknown, are decidedly rare. Conditions at the extremes of the stellar temperature sequence appear to be favorable either to extensive atmospheres or to relatively high spectroscopic excitation of the atmospheres. A large proportion of bright-line spectra are variable, indicating a lack of equilibrium in the outer portions of the stars concerned.

Aside from the general interest attaching to any abnormal or imperfectly understood spectrum, there are at least three specific reasons for studying bright-line stars of Class B, namely, (1) they may assist in the interpretation of novae; (2) bright lines apparently serve as sensitive indicators of conditions in atmospheres of B-type stars; (3) they may teach us new properties of the hydrogen atom.

1. It is possible to select stars of Class Be which form a fairly continuous sequence in the structure of the hydrogen lines from ordinary B stars to novae. Circumstances similar to those causing the characteristic nova spectrum, but less violent, may perhaps be responsible for the unsymmetrical bright lines in certain B-type spectra; but even if this is not the case, it seems evident that the conditions in the atmospheres of several Be stars approach in some degree those prevailing in novae, and we may thus hope to use these stars as an aid in the interpretation of novad phenomena.

2. In many instances Be stars appear to differ from ordinary B-type stars only in the presence of bright hydrogen lines. In some stars the bright hydrogen lines have disappeared without notable changes in other features of the spectrum and without appreciable variation of the total light. These facts make it possible to consider bright hydrogen lines as sensitive indicators of the density or other properties of the atmospheres of the hotter stars, and we are thus offered a method of studying variations too small to be otherwise perceptible.

3. Be stars are also of interest from the viewpoint of general physics, because in them we may observe properties of radiating hydrogen (and some other elements) which differ from the properties shown by laboratory experiments. As examples we may mention the frequent wide doubling of the lines (self-reversal?), and the structure of lines in stars of the P Cygni type, which suggests that in the atmospheres of these stars the hydrogen atoms strongly absorb light

having a wave-length somewhat shorter than each of the Balmer lines, but emit these lines in their normal positions.

I. THE Hα LINE

Historical.—The history of bright-line stars of Class B begins with the discovery by Secchi in 1866 of bright hydrogen lines in the spectra of γ Cassiopeiae and β Lyrae. No additional stars of this kind (with the exception of one or two doubtful instances) were found until the inauguration of the spectroscopic surveys of the Harvard College Observatory by Pickering in 1886. These surveys, made by the aid of the objective prism, produced rich results in this as in other fields of stellar spectroscopy. About 120 B-type spectra were found to have bright hydrogen lines. The Hβ line served most frequently for the detection of hydrogen emission, but in numerous spectra a few of the more refrangible lines were also bright. The portion of the spectrum containing Hα was not shown by these photographs, but in 1894 Campbell, observing visually at Mount Hamilton with the 36-inch refractor and attached spectroscope,[1] found that Hα also is bright. He stated the rule that the intensities of the bright hydrogen lines in B-type spectra decrease from Hα toward the violet.

Ordinary photographic plates are practically insensitive to red light, but the discovery of sensitizing dyes, particularly pinacyanol, has made it possible to prepare emulsions which are quite rapid in the region of Hα, and thus to extend to this part of the spectrum the great advantages of photography. Spectrograms obtained by Merrill[2] in 1912 with the Lick 36-inch refractor showed, in confirmation of Campbell's rule, that in B-type spectra bright Hα is much stronger than Hβ, and that, in spite of the smaller prismatic dispersion in the red, it stands out far more prominently from the background of the neighboring continuous spectrum. Comparison of the appearance of the hydrogen lines in various B-type spectra suggested that numerous stars might exist in which Hα alone is bright. Visual observations by Campbell[3] had, in fact, already shown Hα to be bright in Alcyone, a star in whose spectrum the other hydrogen lines

[1] *Astrophysical Journal,* **2**, 177, 1895.

[2] *Lick Observatory Bulletins*, **7**, 162, 1913.

[3] *Loc. cit.*

are dark.[1] Merrill's observations of the Hα region with slit spectrographs at Mount Hamilton in 1912[2] and at Ann Arbor in 1914[3] of about forty B-type stars resulted in the discovery of six bright-line spectra. A few objective-prism spectrograms of known bright-line stars were obtained in February, 1917, by the same observer with the 2-foot reflector of the Harvard College Observatory, using red-sensitive plates.[4] Bright Hα was clearly shown in the spectra of κ Draconis, β Lyrae, and P Cygni.

The Mount Wilson observations.—It thus became evident that the use of the Hα region of the spectrum would be decidedly advantageous for the detection of hydrogen emission in B-type stars, and it was believed that objective-prism observations comparable with those made by the Harvard College Observatory at Cambridge and Arequipa for the purpose of spectral classification, but showing the Hα region, would result in the discovery of a considerable number of bright-line stars of Class B in addition to those already known. This idea was given a practical trial by Merrill during the years 1919 and 1920 with the 10-inch Cooke refractor of the Mount Wilson Observatory. The preliminary exposures showed that the Cooke telescope with a 15° objective prism was suitable for this work: in particular, that the dispersion (about 440 angstroms per mm at Hα) was sufficient for the purpose, that stars at least as faint as the ninth magnitude could readily be photographed, and that stars over an area about 18° in diameter could be obtained in good focus on a single plate. Several new bright-line stars were soon found, making it highly probable that many others having Hα sufficiently bright to observe were awaiting discovery. A general survey of the Milky Way region was then planned, and special observations of some of the brighter B stars were also included. Humason began active participation in the work at this time, and after September, 1920, made nearly all the objective-prism observations. During the past few months Miss Burwell has given the plates a very careful re-examination and has collected the data for publication.

[1] Later photographs have shown feeble bright portions within the broad absorption at Hβ.

[2] *Loc. cit.*

[3] *Publications of the Astronomical Observatory, University of Michigan*, **2**, 181, 1916.

[4] *Scientific Papers of the Bureau of Standards*, **14**, 487, 1918.

The 10-inch telescope used for the objective-prism observations is of the Cooke "Astrographic" type, having an equivalent focal length of 45.5 inches. The prism, with a circular aperture of 10 inches, has a refracting angle of 15° and causes a deviation of 8°. The linear dispersion at Hα is about 440 A per mm. The lens was so designed that it is achromatic in the blue and violet regions. The color-curve turns sharply away from the lens in the red and more gradually in the ultra-violet. The focal plane of Hα coincides with that for λ 3785 and is 4.1 mm farther from the lens than the focal plane of λ 4200. On our photographs, therefore, only the ends of the spectra are in focus. For this particular type of work a lens having the same back focus for Hα and Hβ[1] would be desirable.

An auxiliary telescope inclined 8° to the axis of the Cooke lens serves for guiding. In a few cases the guiding star has been held at exactly the same point throughout the exposure so that very narrow spectra might be obtained, but in most exposures a small east-and-west motion (parallel to the refracting edge of the prism) was purposely allowed in order to widen the spectra, which aids greatly in detecting a feeble Hα line. Several excellent plates were obtained without any guiding whatsoever. The average exposure time for the large plates is about three hours, but a few exposures have exceeded four hours.

Plates 14×17 inches in size are employed for most of the long exposures. The definition is reasonably good over practically the entire area. Very wide spectra of bright stars are obtained on smaller plates. On moonlight nights the fogging usually caused by the sky light can be avoided by the use of a red color filter; for the exposures on 4×5 plates we often use an Eastman "A" or "F" gelatine filter supported in a cardboard frame immediately in front of the plate-holder.

The emulsions employed are either Seed 23 sensitized by pinacyanol, or Ilford Special Rapid Panchromatic hypersensitized by ammonia. The pinacyanol is applied by the well-known Wallace procedure. The sensitizing bath in which the plates are immersed for four and one-half minutes is made up as follows:

[1] That is, having the same color correction as an ordinary visual objective.

	cc
Distilled water	260
Ethyl alcohol	200
Pinacyanol solution 1:1000	12
Concentrated ammonia	15

This is followed, before drying, by a thorough rinsing in pure ethyl alcohol. The ammoniating bath for the Ilford plates consists of

	cc
Distilled water	375
Ethyl alcohol	125
Concentrated ammonia	13

The subsequent alcohol rinse has usually been omitted. The Ilford plates are perhaps slightly faster in the sense of a lower threshold value, but the Seed 23 plates have somewhat more contrast and finer grain.

Two examples of widened spectra obtained with the ten-inch telescope and objective-prism are shown in Plate XI. (*a*) is copied from a plate taken on January 25, 1922, exposure time $2^h\ 55^m$, emulsion Ilford Special Rapid Panchromatic, hyper sensitized with ammonia. The enlargement is three times. The position of the center of the illustration is R.A. $6^h\ 25^m$, Dec. $+5.^\circ2$ (referring to the red end of the spectrum which lies toward the right). North is toward the right. The brighter stars shown are as follows:

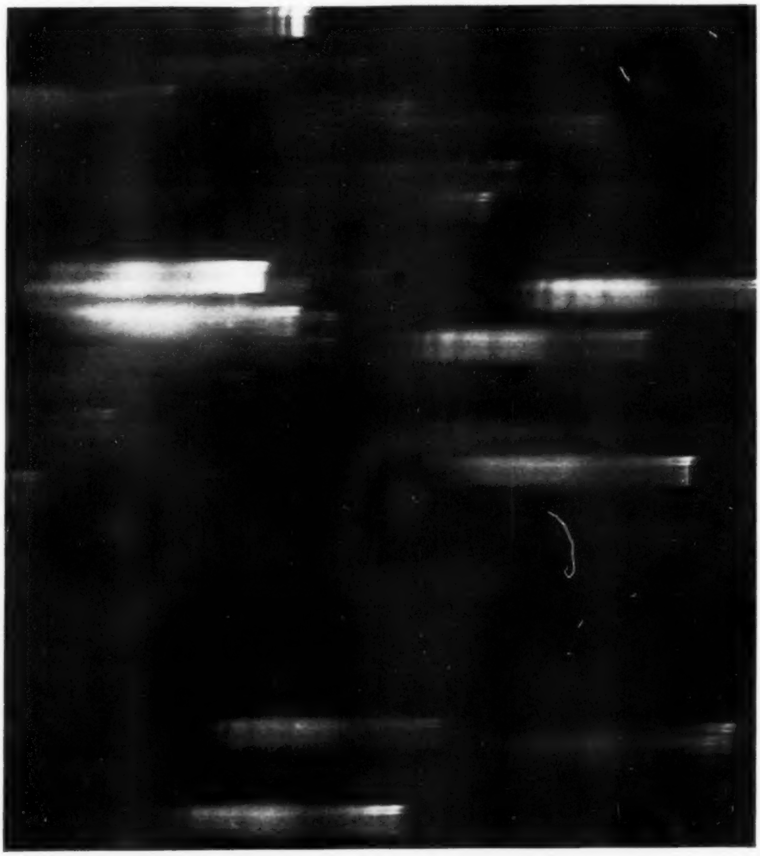
H.D. No.	Mag.	Spect.	Distance of red end from		Remarks
46612	7.1	Mb	Top 2 mm	Left 40 mm	
46241	6.0	Ko	Top 36	Left 35	
46150	6.8	B2	Top 42	Left 39	
46179	6.7	B9	Top 39	Right 0	
46105	6.8	Ao	Top 46	Right 15	
45910	6.7	Be	Bottom 50	Right 9	Bright Hα
45530	7.2	B9	Bottom 15	Right 42	
45431	6.7	Fo	Bottom 3	Right 48	

(*b*) *β* Piscium, 1900 R.A. $22^h\ 58^m.8$, Dec. $+3^\circ\ 17'$; Mag. 4.6, spectrum B5e August 4, 1919, exposure 46 minutes. Emulsion Seed 23 sensitized with pinacyanol. Enlargement five times. The H*α* line is bright, while the ultra-violet hydrogen lines are dark.

Table I contains a list of stars in which we have found the H*α*

PLATE XI

a



b

$H\theta$ $H\alpha$

(a) Portion of an objective-prism spectrogram taken with the ten-inch Cooke telescope. See description on page 394.

(b) Objective-prism spectrogram of β Piscium. See page 394.

TABLE I

DISCOVERY LIST OF STARS HAVING THE $H\alpha$ LINE BRIGHT

No.	H.D.	α 1900	δ 1900	Mag.	Sp.	Ref.	Remarks
1....	108	$0^h\ 0^m9$	$+63°\ 7'$	7.4	O6e	1	
2....	698	0 6.3	+57 39	7.1	B8sea	1	
3....	2789	0 26.3	+66 36	8.2	B2ne	6	
4....	7636	1 11.2	+57 6	7.6	B2ne	1	
5....	B.D.+62°271	1 27.9	+63 7	8.2	B(8)ea	3	Near H.D. 9105
6....	B.D.+54°398	1 45.9	+54 50	8.6	B(2)e	3	
7....	12882	2 1.1	+64 33	7.5	B(2)e	1	
8....	19243	3 0.7	+62 0	6.5	B2e	1	
9....	22298	3 30.3	+54 50	8.4	B2ne	4	
10....	23302	3 39.0	+23 48	3.8	B5nea	7	Electra
11....	23480	3 40.4	+23 39	4.2	B5nea	7	Merope
12....	25348	3 56.6	+53 3	8.2	B(1)e	4	
13....	29866	4 37.3	+40 36	6.1	Bea	2	
14....	30614	4 44.1	+66 10	4.4	O9ea	1	9 Camelop.
15....	33152	5 3.2	+36 53	7.8	B2e	2	
16....	33232	5 3.7	+40 53	8.1	Bepv	2	
17....	33461	5 5.3	+41 6	8.0	B(1)e	3	
18....	33604	5 6.3	+40 5	7.3	B2e	2	
19....	37115	5 31.0	− 5 41	8.2	B(5)e	1	Brighter comp. β.G.C. 2850
20....	37490	5 33.9	+ 4 4	4.5	....	7	ω Orionis B3e
21....	37657	5 35.1	+43 0	7.0	B3ne	2	
22....	37967	5 37.2	+23 10	6.1	B3ne	2	
23....	38191	5 38.9	+21 25	9.5	Be	4	
24....	39340	5 46.9	+26 25	8.1	B3e	3	
25....	B.D.+25°1019	5 47.3	+25 43	8.5	Be	3	
26....	39478	5 47.8	+26 24	8.4	B2ne	4	
27....	41117	5 58.0	+20 8	4.7	B1sea	3	χ^2 Orionis
28....	B.D.+20°1309	6 6.3	+20 7	9.1	Be	3	
29....	43285	6 10.3	+ 6 6	6.0	B(1)e	6	
30....	44637	6 17.7	+15 9	7.7	B3e	3	
31....	45314	6 21.6	+14 57	7.1	B(0)e	3	
32....	45542	6 23.0	+20 17	4.1	....	8	ν Geminorum B5ea
33....	45910	6 25.2	+ 5 57	6.7	Beqv	2	
34....	45995	6 25.6	+11 19	5.8	B2ev	1	
35....	50138	6 46.7	− 6 51	6.6	B8ev	2	
36....	50209	6 47.1	− 0 10	8.3	B(5)e	3	
37....	51354	6 51.9	+18 2	7.1	B4e	.....	
38....	55135	7 6.6	−10 16	7.2	B3ne	2	
39....	55271	7 7.1	−21 38	6.7	Be	2	Brighter comp. β.G.C. 3887
40....	56806	7 13.4	−18 39	9.3	Be	4	

TABLE I—*Continued*

No.	H.D.	α 1900	δ 1900	Mag.	Sp.	Ref.	Remarks
41....	58715	7^h21^m7	+ 8° 29′	3.1	B8neα	8	β Can. Min.
42....	B.D. −13° 2040	7 24.5	−13 34	9.0	B(2)e	4	
43....	59497	7 25.1	−21 38	8.4	B2e	4	
44....	59773	7 26.4	−21 35	8.1	B(3)e	4	
45....	62753	7 40.4	−40 5	6.7	B(2)ne	4	
46....	154218	16 59.0	−36 36	7.7	Be	4	
47....	154243	16 59.2	−36 27	8.3	B2e	5	
48....	154450	17 0.4	−35 37	8.5	Boe	5	
49....	156468	17 12.6	−37 54	8.0	B2e	5	
50....	160095	17 32.9	−33 29	8.7	B8ep	6	
51....	160202	17 33.5	−32 9	6.9	B(1)eβ	5	
52....	161103	17 38.5	−27 12	7.9	Bne	5	
53....	161306	17 39.7	− 9 46	8.3	B(0)e	5	
54....	D.M. −27° 11944	17 41.9	−27 59	9.0	Beq!	5	
55....	163181	17 49.7	−32 27	6.6	B2e	6	
56....	163296	17 50.3	−21 56	6.6	A2e	.....	
57....	163454	17 51.1	−31 0	7.9	B1e	6	
58....	163868	17 53.3	−33 24	7.2	B2e	5	
59....	164906	17 58.3	−24 24	9.0	B(0)e	6	
60....	166566	18 6.1	−15 42	8.1	B2se	5	
61....	166666	18 6.6	−15 36	9.4	B2e	.....	
62....	166734	18 6.9	−10 46	8.3	Beα	.....	
63....	168229	18 13.6	−18 16	9.7	B(1)e	5	
64....	168607	18 15.5	−16 25	8.9	Aose	6	
65....	169226	18 18.6	−12 15	9.1	Be	.....	Brighter comp. β.G.C. 8523
66....	169454	18 19.6	−14 2	6.8	Boe	6	
67....	169515	18 19.9	−12 45	8.3–9.2	Pec	6	RY Scuti
68....	169805	18 21.3	−19 1	8.7	B(1)ne	6	
69....	170061	18 22.4	−14 47	10.6	Be	.....	
70....	172694	18 36.5	−15 57	8.3	Bep	.....	
71....	174105	18 43.8	+15 17	6.9	B8e	.....	
72....	175863	18 52.4	+59 53	6.9	B3e	.....	
73....	180398	19 11.3	+12 56	7.7	B(1)e	.....	
74....	B.D. +14° 3887	19 17.0	+14 42	9.5	Pec	.....	
75....	B.D. +22° 3687	19 20.5	+22 35	8.6	B2e	.....	Brighter comp.
76....	183143	19 23.0	+18 6	6.9	B9seα	.....	
77....	187399	19 44.7	+29 10	7.7	B9eβ	.....	
78....	187567	19 45.4	+ 7 39	6.4	B1e	.....	
79....	B.D. +35° 3950	20 2.0	+35 37	8.8	Be	.....	
80....	192445	20 9.8	+36 1	7.1	B2e	.....	
81....	B.D. +36° 3946	20 10.1	+36 18	8.5	B(0)e	.....	
82....	195407	20 26.0	+36 39	7.7	B1e	.....	
83....	198512	20 45.7	+53 32	8.0	B(2)e	.....	
84....	199218	20 50.6	+40 19	6.5	B5e	1	Brighter comp. β.G.C. 10606
85....	203025	21 14.6	+58 11	6.4	B2e	.....	

TABLE I—*Continued*

No.	H.D.	α 1900	δ 1900	Mag.	Sp.	Ref.	Remarks
86....	207232	21^h42^m4	+50° 13′	7.0	B8e	.....	
87....	209409	21 58.1	− 2 38	4.7	....	7	ο Aquarii B5e
88....	212044	22 16.5	+51 22	7.1	Be	.....	
89....	214748	22 35.1	−27 34	4.2	B8eα	1	ε Pisc. Austr.
90....	224559	23 53.7	+45 51	6.5	B3ne	.....	
	Stars of Classes F and M						
91....	B.D.+61°8	0 4.4	+62 6	9.2	Mep	.....	
92....	42474	6 5.8	+23 14	7.4	Mep	11	
93....	193182	20 13.8	+39 16	6.6	F5e	9	
94....	198287, 8	20 44.2	+38 55	7.0	cF5e	9	
95....	214369	22 32.6	+57 54	8.6–9.3	Mep	10	W Cephei

REFERENCES

1. *Publications of the Astronomical Society of the Pacific*, **32**, 336, 1920.
2. *Ibid.*, **33**, 112, 1921.
3. *Ibid.*, p. 264, 1921.
4. *Ibid.*, **34**, 223, 1922.
5. *Ibid.*, p. 294, 1922.
6. *Ibid.*, p. 351, 1922.
7. *Lick Observatory Bulletins*, **7**, 179, 1913.
8. *Publications of the Astronomical Observatory, University of Michigan*, **2**, 181, 1916.
9. *Publications of the Astronomical Society of the Pacific*, **35**, 263, 1923.
10. *Ibid.* **34**, 59, 1922.
11. *Ibid.*, p. 133, 1922.

NOTES TO TABLE I

Electra, Merope, ω Orionis, and ο Aquarii were found with a slit spectrograph at Mount Hamilton.

ν Geminorum and β Canis Minoris were found with a slit spectrograph at Ann Arbor.

All the remaining stars were found on objective-prism spectrograms taken at Mount Wilson. For dates of observation and other data see Table II.

H.D. 25348: Remark in H.D., "The line Hβ is suspected to be bright."

H.D. 33152: Remark in H.D., "The line Hβ is not seen as a dark line and is suspected to be bright. The other lines are hazy."

H.D. 44637: Remark in H.D., "The line Hβ is not clearly seen and is suspected to be bright."

H.D. 62753: Remark in H.D., "The line Hβ is not distinctly seen and is suspected to be bright."

H.D. 198512: Remark in H.D., "The exact character of the spectrum is not well defined. On the best plates it is nearly continuous with the presence of bright lines suspected. It is apparently of a late division of Class O, or of an early Class B."

H.D. 212044: Remark in H.D., "The presence of a bright Hβ, suspected on the Harvard plate, has been found on photographs taken at Mt. Wilson."

line to be bright.[1] In the second column is given the *H.D.* number (from the *Henry Draper Catalogue*), or the Durchmusterung number in case the star does not appear in *H.D.* The spectral types given in column five are determinations from Mount Wilson slit spectrograms.[2]

The five stars placed by themselves at the end of the table are the only ones falling outside the spectral interval O6 to A2.

Details of the objective-prism observations are collected in Table II. Estimates of the intensity of the bright Hα line are given in the second column on the following scale:

1 = very weak—observable only with widened spectrum and proper exposure
2 = weakest ordinarily observable
3 = medium
4 = strong
5 = very strong—bright Hα strongly exposed even when the adjacent continuous spectrum is very weak

Estimates of the intensity of the dark hydrogen lines in the ultraviolet are given in the third column on the following scale:

0 = absent or barely visible
1 = weak
2 = medium
3 = strong (as in an A0 star)

The chief lines concerned are Hζ, Hη, Hθ, Hι, and Hκ.

In the fourth column, headed "Dates of Observation," are given the last four figures of the Julian Day numbers of representative observations. The material is more extensive than this tabulation indicates, because poor and inconclusive observations have been omitted, and of several observations of the same star made within a few days, only one has been listed in most cases.

Our objective-prism plates show a well-marked bright Hα line in the spectra of most of the "bright Hβ" stars previously announced by other observers. The comparatively few instances in which this is not the case are collected on pages 401 and 402. The spectra of the first five stars have probably changed since the previous observations.

[1] Six stars found at Mount Hamilton or Ann Arbor with slit spectrographs are included for completeness.

[2] See Table V and page 413.

TABLE II

OBJECTIVE-PRISM OBSERVATIONS OF STARS OF CLASS Be

H.D.	Hα	U.V. Hyd.	Dates of Observation*	Remarks
108	3	(0)	2548, 2566	
698	3	(2)	2548, 2569	
2789	3	....	2548	
5394	4	0	2139, 2232, 2580, 2893, 2957	γ Cass.
7636	4	2	2176, 2550, 2586	
B.D.+62°271	4	(0)	2586, 2624	
10516	4	1	2580	ϕ Persei
B.D.+54°398	(1)	....	2524, 2586	
12882	3	(0)	2586, 2623	
19243	3	....	2233, 2578	
20336	3	....	2233, 2578, 2623	
22192	4	(2)	2232, 2579, 2602, 2776	ψ Persei
22298	(3)	0	2730	
23630	1	3	2141, 2175, 2995	Alcyone
25348	3	0	2730, 2776	
25940	4	2	2586, 2776, 3103	c Persei
26906	3	2	2730, 2776	
29866	2	2	2688, 2740	
30614	1−	(1)	2602, 2605, 2624, 2969	9 Camelop.
32343	(2)	(0)	2776	11 Camelop.
33152	3	0	2688	
33232	3	....	2688	
33461	2	1	2688	
33604	3	2	2688	
34921	3	0	2688, 3104	
35345	2	0	2688	
36576	3	1	2728, 2729	
37115	3	....	2586, 2606	
37202	3	2	2362, 2728, 2995	ζ Tauri
37657	2	1	2688	
37967	2	2	2728, 3104	
38010	3	0	2728, 3104	
38191	4	0	2729	
39340	2	0	2729, 3104	
B.D.+25°1019	3	0	2729, 3104	
39478	2	0	2729, 3104	
41117	1−	(1)	2728, 2745	χ^2 Orionis
41335	3	0	2586, 2669, 3080	
B.D.+20°1309	4	0	2729	
43285	2	2	3080	
44637	2	0	2728, 3081	
45314	2	0	2728, 3081	
45725	3	1	2602, 2669	Brightest comp. β Monoc.
45910	3	1	2669, 2687	
45995	3	2	2390, 2728, 3081	
B.D.+10°1172	2	....	2729	N.G.C. 2247
50083	3	1	2669, 3080	
50138	3	2	2669, 3080	
50209	(2)	2	2669	
51354	2	2	3081	
51480	4	0	2700	

* The last four figures of the Julian Day number.

TABLE II—*Continued*

H.D.	Hα	U.V. Hyd.	Dates of Observation	Remarks
55139	3	0	2700	
55271	4	2	2700	
56014	?	1	(2410), (2700)	27 Can. Maj.
56139	2	2	2410	ω Can. Maj.
56806	3	(0)	2700	
57150	4	(0)	2763	ν′ Puppis
58011	(2)	1	2700	
58343	1?	1	(2700)	
B.D.$-13°$2040	(3)	2	2700	
59497	3	1	2700	
59773	(2)	2	2700	
60606	4	1	2763, 2764	z Puppis
62753	3	0	2764	
68980	4	(0)	2763	r Puppis
109387	4	2	2403, 2439, 2518	κ Draconis
127972	3	2	2407, 2475	η Centauri
154218	3	....	2877, 2878	
154243	(3)	....	2877, 2878	
154450	3	....	2877, 2878	
155806	3	0	2877, 2878	
155851	4	....	2877, 2878	
156468	3	....	2877, 2878	
160095	2	....	2877, 2878	
160202	3	....	2540, 2877	
161103	4	....	2541, 2878	
161306	3	(1)	2903	
D.M.$-27°$11944	5!	....	2412, 2546, 2878	
163181	3	0	2539, 2877	
163296	2	3	2903	
163454	3	(0)	2541, 2878	
163868	3	1	2539, 2878	
164906	3	....	2877, 2878	
166566	3	1	2903	
166666	3	0	2903	
166734	1−	1	(2411), (2903)	
167362	4	....	2541, 2878	
168229	3	0	2903	
168607	3	....	2903	
169226	2	....	2903	
169454	3	0	2903	
169515	4	(0)	2903	RY Scuti
169805	3	1	2903	
170061	3	....	2903	
170235	4	....	2546	
172694	3	0	2903	
173219	2	0	2903	
174105	2	2	2938	
174638	4	1	2548, 2556	β Lyrae
175863	3	3	2548	
180398	4	1	2938	
B.D.$+14°$3887	4	....	2587, 2938	
B.D.$+22°$3687	5	....	2587, 2939	
183143	2	0	2587, 2938	
187399	3	1	2939	
187567	3	0	2938	

TABLE II—*Continued*

H.D.	Hα	U.V. Hyd.	Dates of Observation	Remarks
B.D.+35°3950	4	(0)	2961	
190944	3	....	2580, 2961	
191610	3	1	2579, 2961	b² Cygni
192445	3	(1)	2579, 2580	
B.D.+36°3946	2	0	2579, 2961	
193009	3	0	2580, 2939	
193237	4	0	2140, 2580, 2893, 2961	
195407	3	0	2579, 2961	
198512	2	....	2996	
199218	2	2	2579, 2580	
199356	3	0	2579, 2580	
200775	4	....	2487, 2518	N.G.C. 7023
202904	3	2	2580	υ Cygni
203025	2	2	2996	
203374	3	0	2996	
203467	3	0	2487	6 Cephei
206773	4	0	2996	
207232	1	2	2996	
209409	3	(1)	2602	o Aquarii
212044	1	0	2996	
212076	1	3	2995	31 Pegasi
212571	4	1	2602	π Aquarii
214748	1	3	2141, 2175, 2578	ε Pisc. Austr.
217891	3	3	2140, 2175	β Piscium
224559	Rem	(1)	2542, 2567, 2647	
225095	4	0	2542, 2569, 2997	

NOTES TO TABLE II

H.D. 22192, ψ Persei: The ultra-violet hydrogen lines may vary in intensity.

H.D. 29866: The ultra-violet hydrogen lines may vary in intensity.

H.D. 45314: Bright Hα may have increased in intensity between J.D. 2422390 and J.D. 2423081.

H.D. 45910: The ultra-violet hydrogen lines may vary in intensity.

D.M.−27°11944: Bright Hα appears broad but is sharply bounded on the short wave-length side. It probably is of the P Cygni type. This star has an extraordinary spectrum.

H.D. 174638, β Lyrae: Bright λ 6678 He is clearly seen on plates taken on J.D. 2422548 and J.D. 2422556.

H.D. 224559: Bright Hα appears to vary in intensity.

References to bright lines in the spectra of the remaining three objects have been published by other observers, but it is not clear that the present observations constitute evidence of variability.

SPECTRA IN WHICH Hα APPEARS TO VARY

H.D. 9105: See *Publications of the Astronomical Society of the Pacific*, **34**, 180, 1922.

H.D. 58978: Bright Hα is very weak or absent on a plate taken 1921, January 10.

H.D. 181615, υ Sagittarii: The spectrum appears continuous at Hα on a plate taken 1919, July 1. This spectrum is known to be variable.

H.D. 193516: Hα is not seen as a bright line on three plates taken between 1920, September 11, and 1921, September 28.

H.D. 217050: Bright Hα is very weak or absent on a plate taken 1920, November 18. This spectrum is known to be variable.

DOUBTFUL CASES

H.D. 47129: Bright Hα is very weak or absent on several plates taken between 1920, March 6, and 1922, January 22.

H.D. 162586: Hα is not seen as a bright line on several plates taken between 1920, August 2, and 1921, July 7. On two plates it appears dark.

H.D. 193443: Hα is not seen as a bright line on three plates taken between 1920, September 11, and 1921, September 28. A slit spectrogram taken 1924, June 11, shows Hβ, Hγ, Hδ, and Hϵ as absorption lines.

Numerous bright-line spectra of various kinds, including gaseous nebulae, Wolf-Rayet stars, and novae, are shown by our objective-prism photographs, and, as the bright Hα line is an interesting feature in nearly all cases, it seems worth while to make a record of its appearance, especially as the red portion of the spectra of these objects has seldom been observed. In Table III are given the dates of observation and notes chiefly concerning the bright Hα line. For several stars having numerous observations, intermediate dates have been omitted.

The right ascension and declination for 1900 of the objects which are not in the *H.D.* are given below.

T Pyxidis: $9^h0^m.5$, $-31°57'$. See *Publications of the Astronomical Society of the Pacific*, **32**, 200, 1920.

N.G.C. 6334: $17^h13^m.7$, $-35°58'$.

N.G.C. 6357: $17^h18^m.1$, $-34°6'$. Near the Wolf-Rayet star $-34°11675$.

$-29°13988$: $17^h41^m.5$, $-29°57'$. See *Publications of the Astronomical Society of the Pacific*, **33**, 176, 1921.

Trifid nebula: $17^h56^m.6$, $-23°2'$.

Dumbbell nebula: $19^h55^m.3$, $+22°26'$.

Nova Cygni: $19^h55^m.6$, $+53°21'$. See illustration, *Publications of the Astronomical Society of the Pacific*, **32**, 321, 1920.

I.C. 1470: $23^h1^m.0$, $+59°42'$. See *ibid.*, **33**, 176, 1921.

The Hα region of a number of Be stars has been observed with single-prism slit spectrographs attached to one of the large reflectors.

TABLE III

OBJECTIVE-PRISM OBSERVATIONS OF NEBULAE, WOLF-RAYET STARS, AND NOVAE

H.D.	Object	Dates of Observation and Remarks*
37024....	Orion nebula	2605. Hα image strong.
62166....	N.G.C. 2440	2700. Hα image strong.
	T Pyxidis	2436–2441. Hα very strong.
153919....	$-37°11206$	2877. Hα rather weak.
155520....	N.G.C. 6302	2877. Hα image appears stronger than the combined images of N_1 and N_2.
	N.G.C. 6334	2877, 2878. Remarkable group of gaseous nebulae near here.
	N.G.C. 6357	2877, 2878. Hα bright.
	$-29°13998$	2541, 2878. Hα strong.
161944....	N.G.C. 6445	2903. Hα bright.
164270....	$-36°12130$	2540, 2878. Hα strong.
	Trifid nebula	2877, 2903. Hα image differs from the appearance of ordinary photographs.
164740....	N.G.C. 6523	2412, 2877. Interesting detail in Hα image.
165763....	$-21°4864$	2903. λ 5653 extremely strong; λ 5813 very strong.
166449....	N.G.C. 6563	2541, 2878. Hα bright.
166468....	N.G.C. 6565	2541, 2878. Hα strong.
166802....	N.G.C. 6572	2845. Hα very strong.
167276....	Nova Ophiuchi	2550. Hα very strong.
168206....	$-11°4953$	2903. Hα probably bright but not strong.
168520....	Omega nebula	2411, 2903. Much fine detail in Hα image.
170124....	Gaseous nebula	2877, 2878. Hα bright.
174107....	Nova Aquilae	2140, 2524, 2542, 2845. Hα very strong. On last plate continuous spectrum strong in violet and weak in red.
175353....	Ring nebula	2546. Hα strong
184738....	$+30°3639$	2939. The combined image of Hα and λλ 6548, 6583 is extremely strong.
186924....	N.G.C. 6826	2487. Hα bright.
	Dumb-bell nebula	2938, 2939. Interesting detail in Hα image. Fairly strong image in ultra-violet, perhaps λ 3727.
	Nova Cygni	2566, 2570, 2602, 2623. Hα very strong.
191765....	$+35°4001$	2579, 2961. Hα strong.
191916....	N.G.C. 6884	2961. Hα bright.
192103....	$+35°4013$	2579, 2961. Hα strong.
192163....	$+37°3821$	2579, 2961. Hα strong.
192641....	$+36°3956$	2579. Hα bright.
193077....	$+36°3987$	2579, 2961. Hα bright.
193576....	$+38°4010$	2579, 2961. Hα bright.
193793....	$+43°3571$	2487, 2579, 2961. Hα bright.
201272....	N.G.C. 7027	2580. The combined image of Hα and λλ 6548, 6583 is very strong.
	I.C. 1470	2997. Hα strong.

* The last four figures of the Julian Day numbers of the dates of observations are given. The remarks refer entirely to emission lines.

The dispersion at Hα is about 170 angstroms per millimeter. The results are collected in Table IV.

II. THE BLUE-VIOLET SPECTRAL REGION

The objective-prism spectrograms obtained with the 10-inch telescope yield little information concerning the lines Hβ, Hγ, and Hδ, as the part of the spectrum containing them is much out of focus. We have observed this spectral region, therefore, with single-prism slit spectrographs attached to one of the large reflectors. The program included all the stars found with the 10-inch telescope to have the Hα line bright, and in addition a number of previously known "bright Hβ" stars, particularly those for which there is no published record of observations with a slit spectrograph. An 18-

TABLE IV

OBSERVATIONS OF Hα WITH SLIT SPECTROGRAPHS

H.D.	Object	Date	Character of Hα
698	....	1923 February 6	Strong bright
23302	Electra	1919 November 8	Weak bright on absorption
23480	Merope	1919 November 8	Very weak bright on absorption
23630	Alcyone	1919 November 8	Strong bright
23862	Pleione	1919 November 8	Absorption
		1921 November 12	Absorption
24534	X Persei	1923 September 1	Strong bright
29866	....	1923 February 5	Strong bright
30614	9 Camelop.	1921 February 28	Very weak bright
		1924 March 12	Very weak bright
		March 13	Very weak bright
41117	χ^2 Orionis	1921 February 28	Weak bright
		March 30	Weak bright
51354	....	1923 January 7	Strong bright
58715	β Can. Min.	1921 March 29	Bright
109387	κ Draconis	1924 March 12	Strong bright
155806	....	1923 April 30	Very strong bright
166734	....	1922 October 8	Bright. Not sharp
183143	....	1922 October 7	Probably bright
		October 8	Probably bright
		1923 September 1	Bright
214748	ϵ Pisc. Austr.	1921 November 12	Strong bright

NOTES TO TABLE IV

H.D. 166734: The continuous spectrum is strong in the red.

H.D. 183143: The continuous spectrum is very strong in the red. The first two plates are overexposed.

inch camera was used in practically every exposure, the dispersion being as follows: at $H\beta$, 57 A per mm; at $H\gamma$, 36; and at $H\delta$, 28. The data from these plates are collected in Table V and the accompanying notes.

In the first column is given the *H.D.* number (from the *Henry Draper Catalogue*), or the Durchmusterung number in case the star does not appear in *H.D.* The common names of a few bright objects, and, for double stars, the numbers in Burnham's *General Catalogue of Double Stars* (β.G.C.), are added in the notes. Descriptions of the hydrogen lines are found in the next three columns. The character of the lines (Ch.) is described by means of the following symbols:[1]

A = absorption
C = continuous
P = P Cygni type, i.e., a bright line with a dark line on its short wave-length edge.
S, D refer to bright components
S = single
D = double

The intensities (Int.) refer only to the bright components and are on the following basis:

0.5 = very weak
1 = weak
2 = medium
3 = strong
4 = very strong

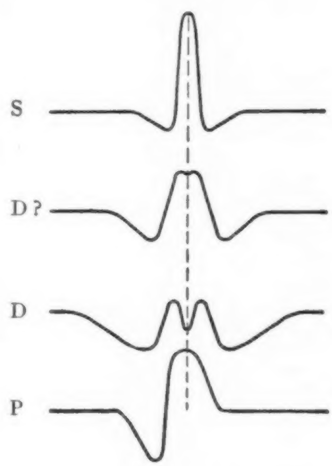


FIG. 1.—Typical intensity curves of hydrogen lines.

The bright lines are nearly all superposed on wider dark lines. Typical intensity-curves of the different types of lines are shown in Figure 1. The structure of a complex line is usually symmetrical about the normal position of the line, but there are a few exceptions. Aside from lines of the P Cygni type, which are of course decidedly unsymmetrical, there are complex lines of the ordinary type in which the two bright components are of unequal intensity. All definite instances of this kind are mentioned in the notes following Table V.

[1] The notation is similar to that employed by Lockyer, *Monthly Notices*, **84**, 409, 1924.

TABLE V

DATA FROM SLIT SPECTROGRAMS OF THE BLUE AND VIOLET REGION

H.D.	Hβ		Hγ		Hδ		Type	Radial Vel.
	Ch.	Int.	Ch.	Int.	Ch.	Int.		
108	S	3	S	3	S	2	O6e	−60.
698	A	...	A	...	A	...	B8sea	Variable.
2789	C	...	A	...	A	...	B2ne	
7636	S	3	S	2	S	0.5	B2ne	−15.
9105	A	...	A	...	A	...	B4s	−43:
+62°271	S	1	A	...	A	...	B(8)eβ	
+54°398	S	1	C	...	C	...	B(2)e	
12882	D?	1	C	...	A	...	B(2)e	
15325	A	...	A	...	A	...	B1	−34.
15450	S	3	S	2	S	1	B(0)ne	
19243	S	2	S	1	C	...	B2e	−29:
22298	D?	0.5	C	...	A	...	B2ne	
23862	A	...	A	...	A	...	B8n	
24534	D?	1	C	...	C	...	B(0)e	
25348	D?	1	D	0.5	C	...	B(1)e	
29866	A	...	A	...	A	...	Bea	
30614	A	...	A	...	A	...	O9ea	
33152	S	3	S	2	S	1	B2e	−4.
33232	...	...	...	...	...	...	Bepv	
33461	D	0.5	A	...	A	...	B(1)e	
33604	S	2	S	1	S	0.5	B2e	
34921	D?	2	D?	1	C	...	B(0)e	
36576	D?	1	C	...	A	...	B1ne	
37115	D	3	D	1	A	...	B(5)e	
37657	D?	1	C	...	A	...	B3ne	
37967	S	1	A	...	A	...	B3ne	
38010	D?	2	D	1	C	...	Be	
38191	D?	2	D	1	C	...	Be	
39340	D?	2	C	...	A	...	B3e	
+25°1019	S	2	D?	0.5	C	...	Be	
39478	D?	1	D	0.5	A	...	B2ne	
41117	A	...	A	...	A	...	B1sea	
+20°1309	S	2	C	...	A	...	Be	
43285	D?	0.5	A	...	A	...	B(1)e	
44637	S	2	C	...	...	...	B3e	
45314	D?	1	C	...	C	...	B(0)e	
45677	...	...	...	...	...	...	Bep	
45910	P	...	P	...	P	...	Beqv	
45995	...	...	...	...	...	...	B2ev	
50083	D?	2	D	1	C	...	B1e	
50138	...	...	...	...	...	...	B8ev	
50209	S	1	A	...	...	...	B(5)e	
51354	C	...	A	...	A	...	B4e	
51480	P	4	P	3	P	2	B8eq	
55135	S	2	A	...	A	...	B3ne	
55271	D?	1	D	0.5	...	...	Be	
56806	D?	2	C	...	C	...	Be	
−13°2040	S	3	S	1	D	0.5	B(2)e	
59497	D?	2	D?	0.5	A	...	B2e	
59773	D?	1	A	...	...	...	B(3)e	

TABLE V—*Continued*

H.D.	Hβ Ch.	Hβ Int.	Hγ Ch.	Hγ Int.	Hδ Ch.	Hδ Int.	Type	Radial Vel.
62753	D?	2	D	0.5	A	....	B(2)ne	
65875	S	3	D?	1	C	....	B3e	
120324	A	....	A	....	A	....	B2	
127972	A	....	A	....	A	....	B(0)n	
152236	P	1	P	1	A	....	B(2)eq	
154218	D?	2	D	0.5	....	....	Be	
154243	S	2	D?	1	....	....	B2e	
154450	S	3	S	2	....	....	B0e	See note.
155806	S	1	C	....	....	....	O8e	
156468	S	2	S	1	....	....	B2e	See note.
160095	....	....	....	....	....	....	B8ep	
160202	D?	1	A	....	....	....	B(1)eβ	
161103	D?	2	D	1	....	....	Bne	
161114	....	....	....	....	....	....	Bep!	
161306	S	3	D?	2	C	....	B(0)e	
−27°11944	P	4	P	3	....	....	Beq!	
163181	....	....	....	....	....	....	B2e	Variable.
163296	....	....	....	....	....	....	A2e	
163454	S	2	D?	1	....	....	B1e	
163868	S	2	A	....	A	....	B2e	
164906	S	1	C	....	C	....	B(0)e	
166566	S	1	A	....	....	....	B2se	
166666	D?	1	C	....	....	....	B2e	
166734	A	....	....	....	....	....	Beα	
168229	S	2	D?	0.5	....	....	B(1)e	
168607	S	3	S	1	....	....	A0se	
169226	P	2	....	....	....	....	Be	
169454	P	1	A	....	....	....	B0e	
169515	S	3	S	2	....	....	Pec	
169805	D?	2	D	1	....	....	B(1)ne	
170061	S	2	C	....	....	....	Be	
172694	....	....	....	....	....	....	Bep	
174105	D?	1	A	....	A	....	B8e	
175863	D?	1	A	....	A	....	B3e	−27.
180398	D?	1	A	....	A	....	B(1)e	
+14°3887	S	3	S	2	....	....	Pec	
+22°3687	S	3	D	2	....	....	B2e	−3.
183143	A	....	A	....	A	....	B9seα	+13.
187399	S	1	A	....	A	....	B9eβ	
187567	S	2	C	....	A	....	B1e	See note.
+35°3950	S	2	D?	1	C	....	Be	
192445	S	2	C	....	A	....	B2e	
+36°3946	S	2	D	0.5	C	....	B(0)e	
193009	S	1	D	0.5	C	....	B1ne	
195407	D	....	D	....	A	....	B1e	
198512	S	2	D?	1	C	....	B(2)e	
199218	D	1	A	....	A	....	B5e	
199356	D?	2	D	0.5	C	....	Be	
203374	S	2	S	1	C	....	Be	
206773	D	2	D	1	C	....	Be	
207232	D	0.5	A	....	A	....	B(8)e	

TABLE V—*Continued*

H.D.	Hβ Ch.	Hβ Int.	Hγ Ch.	Hγ Int.	Hδ Ch.	Hδ Int.	Type	Radial Vel.
207757	S	5	S	4	S	3	Bepv	
212044	S	2	D?	1	C	....	Be	See note.
214748	A	....	A	....	A	....	B8ea	
224559	D	2	D	1	A	....	B3ne	0.
225095	S	3	S	2	D?	1	B1e	

DATES OF OBSERVATION AND NOTES

H.D. 108: 2596, 2624, 2625, 2914, 3245, 3629. The hydrogen lines from Hβ to Hε consist of rather narrow bright lines superposed on faint dark lines. The dark components on the violet sides of the bright lines are slightly stronger than those on the red sides, and as they are near the positions of the lines which belong with the Pickering series, they are probably due in part to He+. The Pickering series appears wholly dark; the following lines are seen: λλ 4025.64?, 4199.87, 4541.63, and 5411.57. The velocity from the bright lines Hβ, Hγ, and Hδ is −58, and from the dark Pickering lines λλ4199.87 and 4541.63, −63 km/sec. These values are in substantial agreement with Plaskett's results in *Monthly Notices of the Royal Astronomical Society*, **84**, 84, 1923. λ4686 He+ is bright. D3 and λ4471 He are bright, the latter with a dark companion on the violet side. λ4552 Si is bright and there is a trace of bright λ4567 Si. There are several bright lines in the region λ 4640–50. H and K are fairly strong dark lines. This is a very interesting spectrum, well worth further study.

H.D. 698: 2626, 3244, 3245, 3809. Spectroscopic binary. Velocities from individual plates are +58, (+29), +20, and −10 km/sec., respectively.

H.D. 2789: 2916, 3339.

H.D. 7636: 2596, 3810. Sharp H and K.

H.D. 9105: 2594, 2624, 2746, 2925, 2927, 3245. This spectrum is evidently variable, as the bright hydrogen lines have disappeared since the Harvard observations. See *Publications of the Astronomical Society of the Pacific*, **34**, 180, 1922. The velocities from individual plates range from −29 to −60 km/sec., but the mean value, −43, agrees with that reported by Plaskett in *Monthly Notices of the Royal Astronomical Society*, **84**, 84, 1923.

B.D.+62°271: 2897.

B.D.+54°398: 2667, 2907. Aside from bright Hβ, the spectrum is nearly continuous.

H.D. 12882: 2719, 2917, 3988. The bright line at Hβ is stronger on the violet side.

H.D. 15325: 3307. In Praesepe cluster. Brighter component of β.G.C. 1277. The bright lines noted by Harvard seem to have disappeared.

H.D. 15450: 4018. Observed with 10-inch camera.

H.D. 19243: 2596, 2951, 3335, 3652, 3778. Sharp H and K.

H.D. 22298: 2926.

H.D. 23862, Pleione: 2271, 3006, 3339, 4046. The first two plates include Hα, which is a strong absorption line. The Balmer series, once partially bright, now consists of broad, strong, dark lines, as in 1912 (*Lick Observatory Bulletin*, **7**, 167, 1913).

H.D. 24534, X Persei: 2719, 3064, 3664. Sharp H and K. The dark lines, aside from H and K, are faint and indistinct.

H.D. 25348: 2926. H and K are probably present as weak, narrow, dark lines. The other dark lines are indistinct. Remark in H.D., "The line Hβ is suspected to be bright."

H.D. 29866: 2689, 2695, 3335. Aside from the hydrogen lines, the spectrum is nearly continuous.

H.D. 30614, 9 Camelopardalis: 2631. Sharp H and K. See description of spectrum by Lee, *Astrophysical Journal*, **37**, 1, 1913.

H.D. 33152: 2689, 3360, 3806, 3808. The bright hydrogen lines are unusually narrow. Remark in H.D., "The line Hβ is not seen as a dark line and is suspected to be bright. The other lines are hazy." On our plates the helium lines are distinct.

H.D. 33232: This variable spectrum is under observation and will be described in a future *Contribution*.

H.D. 33461: 2694, 4047. The violet component of bright Hβ is the stronger. The edges of Hγ are probably faintly bright.

H.D. 33604: 2690, 3335, 3456, 3908. The bright hydrogen lines seem to have become weaker since the first observation. On the last photograph the bright components of Hγ and Hδ are weak and indistinct.

H.D. 34921: 2973, 3692.

H.D. 36576, 120 Tauri: 2973.

H.D. 37115, brighter component of β.G.C. 2850: 2626. The measured separations of the bright components are: Hβ, 3.8 A; Hγ, 3.1 A.

H.D. 37657: 2694.

H.D. 37967: 2746.

H.D. 38010: 3653. Aside from the hydrogen lines, the spectrum is nearly continuous.

H. D. 38191: 2969. Aside from the hydrogen lines, the spectrum is nearly continuous.

H.D. 39340: 2759, 3338, 3808. The plates apparently show a slight change in the bright Hβ line: on the first plate the component of longer wave-length is the stronger, but on the other plates the two components are of nearly equal intensity. The measured separations of the bright components of Hβ on the three plates are respectively, 4.2, 3.2, and 3.1 A.

B.D.+25°1019: 2779, 2790, 2981.

H.D. 39478: 2953.

H.D. 41117, χ^2 Orionis: The orbit of this spectroscopic binary (*Lick Observatory Bulletins*, **6**, 144, 1911) is under investigation by Mr. Sanford.

B.D.+20°1309: 2779, 3782.

H.D. 43285: 3127, 3336, 3809.

H.D. 44637: 2748.

H.D. 45314: 2747, 3692. Sharp H and K. The other lines are very faint.

H.D. 45677: This peculiar spectrum is under observation and will be described in a future *Contribution*.

H.D. 45910: The remarkable variations in this spectrum have been described by Plaskett, *Publications of the Astronomical Society of the Pacific*, **35**, 145, 1923, and by Merrill, *ibid*., p. 303, 1923.

H.D. 45995, brighter component of β.G.C. 3427: The structure of the bright hy-

drogen lines varies. This spectrum is under observation and will be described in a future *Contribution*.

H.D. 50083: 3725. Measured separation of the bright components of Hγ, 1.8 A.

H.D. 50138: The structure of the bright hydrogen lines varies in a period of about twelve days. This spectrum is under observation and will be described in a future *Contribution*.

H.D. 50209: 2694.

H.D. 51354: 3126, 3134, 3337. Trace of a double bright line at Hβ.

H.D. 51480: This spectrum, which is of the P Cygni type, is under observation and will be described in a future *Contribution*.

H.D. 55135: 2720. There is probably a very weak double bright line superposed on the broad absorption line at Hγ.

H.D. 55271, brighter component of β.G.C. 3887: 2718.

H.D. 56806: 2981.

B.D. $-13°2040$: 2979.

H.D. 59497: 2980.

H.D. 59773: 3038.

H.D. 62753: 3036, 3810. On the first plate the violet components of bright Hβ and Hγ are stronger than the red components. The structure of Hβ may vary, as the bright components of Hβ appear to be more nearly equal in intensity on the second plate than the first. The second plate is under-exposed at Hγ.

H.D. 65875: 3039, 3725.

H.D. 120324, μ Centauri: 3951. The recent disappearance of bright hydrogen lines from this spectrum, noted by several observers, is confirmed by our photograph.

H.D. 127972, η Centauri: 3951. This spectrum is known to vary.

H.D. 152236, ζ' Scorpii: 3928.

H.D. 154218: 2897.

H.D. 154243: 2897. This star is very near the preceding star, H.D. 154218.

H.D. 154450: 3248, 3927, 3986. The apparent radial velocities from the bright Hβ and Hγ lines for the three plates are $+24$, $+2$, and $+7$ km/sec., respectively. The divergence of the first plate from the other two probably indicates a change in the structure of the lines or in the radial velocity of the star.

H.D. 155806: 3540.

H.D. 156468: 3220, 3951. The displacement of the narrow bright Hβ line corresponds to a velocity of about -50 km/sec., but it is not certain that this is the actual radial velocity of the star.

H.D. 160095: 3245. The hydrogen lines are peculiar. This spectrum will be described in a future *Contribution*, after further observation.

H.D. 160202: 2769, 3664.

H.D. 161103: 2914. The measured separation of the bright components of Hγ is 2.8 A.

H.D. 161114: The spectrum of this "iron" star is under observation and will be described in a future *Contribution*. See *Publications of the Astronomical Society of the Pacific*, **36**, 225, 1924.

H.D. 161306: 2913.

D.M. $-27°11944$: This P Cygni-type spectrum is under observation and will be described in a future *Contribution*.

H.D. 163181: 2914, 3247, 3605, 3960. Hβ, Hγ, and Hδ are chiefly absorption lines,

but at Hβ and possibly at Hγ there appear to be indistinct bright edges. Spectroscopic binary. Approximate velocities from the last three plates are $+90$, $+116$, and -226 km/sec., respectively.

H.D. 163296: The structure of the hydrogen lines is very peculiar and appears to vary. This spectrum is under observation and will be described in a future *Contribution*.

H.D. 163454: 3190, 3928, 3987.

H.D. 163868: 2926, 3960.

H.D. 164906: 3307.

H.D. 166566: 2917.

H.D. 166666: 3221, 3976.

H.D. 166734: 3245, 3927, 3947.

H.D. 168229: 3220.

H.D. 168607: 2952, 3951.

H.D. 169226, brighter component of β.G.C. 8523: 3664, 3960, 3987. All the spectrograms are under-exposed.

H.D. 169454: The spectrum of this star, which contains strong dark lines of He, Si, and O, will be described in a future *Contribution*.

H.D. 169515, RY Scuti: This peculiar spectrum, in which a bright line is seen in the position of the nebular line λ 4658, has been described in the *Publications of the Astronomical Society of the Pacific*, **34**, 134 and 295, 1922. A more complete description will be published in a future *Contribution*.

H.D. 169805: 2979. The measured separation of the bright components of Hγ is 3.2 A.

H.D. 170061: 3244. Plate under-exposed.

H.D. 172694: This peculiar spectrum, which somewhat resembles that of H.D. 33232, is under observation and will be described in a future *Contribution*.

H.D. 174105: 2951, 3894.

H.D. 175863: 2566, 2567, 2570, 3191, 3984. The bright portions of Hβ appear weaker on the last two plates than on the others.

H.D. 180398: 2951, 3986. The bright portion of Hβ may be weaker on the second plate than on the first.

B.D.$+14^\circ$3887: This spectrum, which contains some unusual bright lines, is under observation and will be described in a future *Contribution*.

B.D.$+22^\circ$3687, brighter component of β.G.C. 9287: 2652, 2956, 3951, 3976. The measured separation of the bright components of Hγ is 2.3 A.

H.D. 183143: 2951, 3338, 3988. Remark in H.D., "The spectrum appears to be nearly continuous." On our plates several lines are strong and well defined, notably Hγ, λ 4471, and λ 4481. Thus it is possible that the spectrum has changed since the Harvard photographs were taken.

H.D. 187399: 3895.

H.D. 187567: 2953, 3336, 3337, 4015. Sharp H and K, measured on three plates, give a velocity of -2 km/sec. The other lines do not yield reliable results.

B.D.$+35^\circ$3950: 2977, 3960. Sharp H and K.

H.D. 192445: 2834, 3984.

H.D. 193009: 2951, 3896.

H.D. 195407: 2952, 3897, 3930, 3952. The radial velocities derived from the hydrogen lines of the first, second, and fourth plates are -68, -82, and -82 km/sec. re-

spectively, giving a mean value of -77 km/sec. The intensities of the bright portion of the hydrogen lines vary.

H.D. 198512: 3307, 3928.

H.D. 199218, brighter component of β.G.C. 10606: 2853, 2925, 3247, 3896. Measured separation of bright components of $H\beta$ (mean value from three plates) 3.9 A.

H.D. 199356: 2973. The separation of the bright components of $H\gamma$ is not accurately measurable on this plate, but is approximately 4.2 A. Sharp H and K.

H.D. 203374: 3013. Sharp H and K.

H.D. 206773: 2955, 3947. The measured separation of the bright components of $H\beta$ is 4.0 A; of $H\gamma$, 4.3 A. Sharp H and K. Aside from the hydrogen and calcium lines, the spectrum is very nearly continuous.

H.D. 207232: 3039, 3926. The measured separation of the bright components of $H\beta$ on the second plate is 4.0 A.

H.D. 207757: This spectrum, which has shown a remarkable change in the past few years (see *Harvard College Observatory Bulletin*, 762) is under observation and will be described in a future *Contribution*.

H.D. 212044: 3014, 3984. Approximate radial velocity from sharp H and K, -16 km/sec. Results from the other lines are not reliable.

H.D. 214748, ϵ Piscis Australis: 2179, 2576.

H.D. 224559: 2596, 2656, 2953, 3604, 3779. Measured separation of the bright components of $H\beta$, 3.1 A; of $H\gamma$, 3.1 A; of $H\delta$, 2.8±. The bright portions of $H\delta$ are feeble and indistinct; it is chiefly an absorption line. Sharp H and K.

H.D. 225095: 2955, 3952.

It is impossible wholly to eliminate photographic effects from the classification of the character of the hydrogen lines. Better photographs might in some instances change the description from D? to D, or even, in the case of very faint bright components, from C or A to D. In this connection, however, it must be borne in mind that the symbols C and A may include lines in which bright portions probably exist, provided the bright components exhibit a degree of distinctness less than that required for the intensity 0.5. Photographic conditions such as the width of the spectrum or the effective contrast of the emulsion might, of course, alter one's judgment on this point. In view of the fact that the bright lines of these stars are subject to change, and because it is in the weak lines that changes may first become obvious, it will perhaps be worth while to describe exactly the significance we have attached to the intensities 1 and 0.5. A line of intensity 1 is a clearly marked bright line, seen at a glance, but of the lowest intensity at which a reliable setting can be made with the measuring-microscope (magnifying-power 26). Intensity 0.5 denotes a line fainter than 1 but of whose reality one becomes

convinced after careful inspection. Doubtful bright lines have been omitted from Table V.

The spectral types are those estimated from the Mount Wilson plates. For types earlier than B0 the system suggested by H. H. Plaskett[1] has been employed. When the spectral subdivision is uncertain, it is either omitted or placed in parentheses. The following additional symbols recommended at the Rome meeting of the International Astronomical Union[2] have proved convenient:

e = emission lines
n = diffuse lines
s = sharp lines
p = peculiar
v = variable
q = lines of the P Cygni type

The bright parts of the hydrogen lines grow weaker from Hα toward the violet in all cases. The letters α and β indicate which is the last line to exhibit bright components. In many stars the transition along the series from bright lines to wholly dark lines is so gradual that it is impracticable to specify the line at which it occurs.

III. DISCUSSION

Table I shows that our observations have added 90 early-type stars to the list of those known to possess hydrogen emission lines. Eighty-four of these discoveries have resulted from photographing the Hα region with the 10-inch telescope and objective prism. These observations are largely confined to the Milky Way north of declination $-40°$. The areas covered by plates having fairly good definition and about the standard effective exposure are charted in Figure 2. Within these areas 60 bright-line stars[3] were previously known, and 80 additional ones[4] have been found from our objective-prism observations. The Harvard spectrographic survey showed 49 bright-line stars in the same areas.

[1] *Publications of the Dominion Astrophysical Observatory*, 1, 366, 1922.

[2] *Transactions of the International Astronomical Union*, 1, 95, 1922.

[3] This includes two stars listed in Table I which were previously detected by Merrill from slit spectrograms.

[4] Not counting χ^2 Orionis and 9 Camelopardalis, in which the bright Hα line is too weak to show on our regular plates.

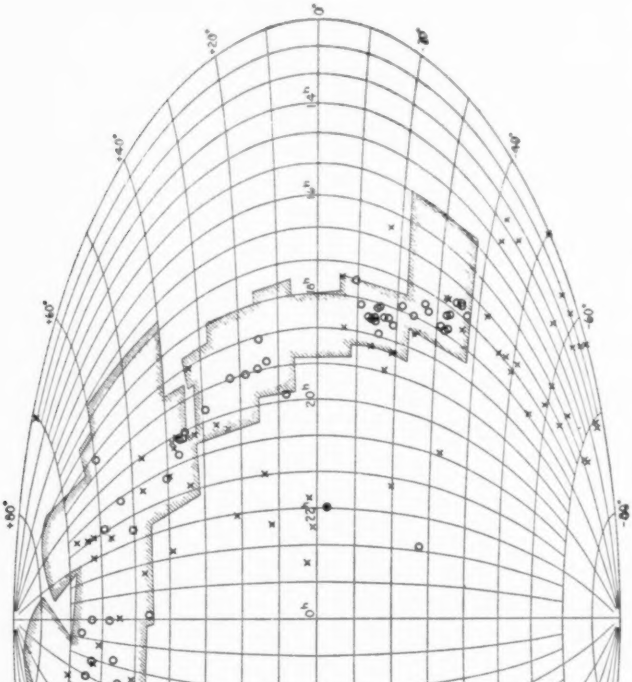


FIG. 2.—Chart of stars of class Be. Circles represent those discovered in the present investigation; crosses those previously known. Six stars found by one of the writers with slit spectrographs at Mount Hamilton or Ann Arbor are marked by crosses within circles. The areas indicated by shading were covered by the Mount Wilson objective-prism observations.

In 41 of our 84 stars, the bright Hβ line is so weak that it could scarcely have been detected on the Harvard plates. This illustrates the advantage of working with the Hα line. The average intensity of the Hβ line in the remaining 43 stars is probably nearly the same as in those discovered at Harvard, and some explanation of the non-appearance of the bright lines on the Harvard plates must be sought. Eight stars are not in the *Henry Draper Catalogue*, leaving 35 which were classified at Harvard. In a few cases the bright lines may have been missed at Harvard because of unfavorable observing circumstances, but the number appears too great to be accounted for entirely in this manner, and the question arises whether some of our additional stars are not ones in which the bright hydrogen lines have come into existence, or at least have become more intense, since the epoch of the earlier observations. Such an explanation is in harmony with the fact that variations in the intensity of the bright hydrogen lines are known to occur in numerous Be stars. A re-examination of the Harvard plates and comparison of the appearance of the hydrogen lines with the data in Table V might throw considerable light on this problem. A better attack would be the repetition of some of the Harvard observations or our own after five years or more have elapsed.

Several stars have experienced a decline in the intensity of the bright hydrogen lines in recent years, notable examples being Pleione and μ Centauri. The list on pp. 401 and 402 adds a few stars to those previously recorded.

It would be desirable to ascertain whether stars of class Be are approximately in statistical equilibrium, that is, whether in a given interval of time the B stars *entering* the bright-line stage are about as numerous as those *leaving* it. We should expect this to be true, but until recently the only known instances of progressive change were those in which the bright lines were growing weaker. This may have been caused partly by the fact that bright-line stars are more closely watched than ordinary stars and hence changes in them are more likely to be detected. A few instances of the appearance of bright lines or of an increase in their strength have, however, been announced by Miss Cannon[1] and by Perrine,[2] and the present

[1] *Harvard Bulletin*, No. 779, 1922. [2] *Popular Astronomy*, **27**, 91, 1919.

observations may contain numerous examples of the same phenomenon. It will not be many years before this matter can be discussed much more adequately than at present, because of the increased attention being devoted to the Be stars by a number of observers. This remark also applies to a related question, namely, whether a B-type star can pass into the bright-line stage more than once.

Distribution.—Figure 2 shows the distribution of all known Be stars. There is a tendency for them to fall into groups which in nearly every instance lie near the center line of the Milky Way. The strong general condensation near the galaxy is evident from the chart. The material at hand is not suitable for an exact study of this phenomenon.

Four groups of Be stars are in regions which are also rich in Wolf-Rayet stars. The approximate centers of these regions are:

R.A.	Dec.	
5^h20^m	$-68°$	Greater Magellanic cloud
10 30	-58	
20 10	$+36$	
17 30	-35	

The last three regions are near the center line of the Milky Way.

Why should Wolf-Rayet and Be stars be concentrated in the same regions of space? Are the conditions in these regions such as to provoke emission lines in stars passing through them, or have the stars therein a common origin which favors emission lines?

The distribution of the bright-line stars in Table V among the various spectral subdivisions is as follows:

Class	No. in Table V	Class	No. in Table V
O6	1	B5	3
O8	1	B8	9
O9	1	B9	2
B0	10	A0	1
B1	16	A2	1
B2	23	B	24
B3	9	Pec	2
B4	2		

The relatively high frequency of emission lines among classes B0 to B3, inclusive, is evident. It becomes still more striking when

compared with the total number of stars in corresponding groups. We may use for this purpose the Harvard counts of the stars brighter than 8.25 in the *Henry Draper Catalogue.*

Class	All	No. in Table V	Ratio
B0–B5	2061	63	1:N
B8	1604	9	1:5N
B9	2752	2	1:40N
A0	6320	1	1:200N

The actual values of the ratio are not of interest, because Table V contains but a small part of the total number of bright-line stars. These stars do, however, appear to represent a fair sample of the whole as far as spectral classes are concerned, and the relative proportions constitute direct evidence of the preference of bright-line stars for classes B0 to B5.

MOUNT WILSON OBSERVATORY
November 1924

TWO STELLAR SPECTRA OF THE P CYGNI TYPE[1]

By PAUL W. MERRILL

ABSTRACT

H.D. 51480.—The bright lines were discovered at Harvard about 1892, and recent observations at Mount Wilson have shown them to be bordered on their violet sides by dark lines as in P Cygni. The hydrogen lines are very similar to those of P Cygni; silicon, nitrogen, and helium lines are weaker; and enhanced iron lines are stronger. The general appearance of the spectrum remained almost unaltered from 1919 to 1924, but measurements show certain changes in the positions of the lines.

DM. $-27°11944$.—Bright hydrogen lines were found at Mount Wilson in 1920, and subsequent observations have shown this spectrum to possess the P Cygni characteristics in a more marked degree than any star previously studied. The bright hydrogen lines are extremely intense, and all the lines are very wide. In these respects the star occupies a position intermediate between P Cygni and a typical nova.

The distinguishing features of the spectrum of P Cygni are strong bright lines bordered on their violet sides by dark lines. Detailed descriptions have been published by several observers.[2] This object appeared as a new star in 1600, and the fact that its spectrum still retains a partial resemblance to that of a typical nova is pertinent to the interpretation of the phenomena of novae. A short list of additional stars of the P Cygni type was published some years ago by Miss Cannon,[3] but thus far, aside from P Cygni itself, only one, B.D. $+11°4673$, has been described in detail.[4]

The two following stars have been found at Mount Wilson to possess the P Cygni characteristics:

H.D.	DM.	α 1900	δ 1900	Mag.
51480	$-10°1774$	$6^h\ 52^m4$	$-10°\ 41'$	7.0
	$-27°11944$	17 41.9	-27 59	9.0

H.D. 51480

The discovery of bright lines in the spectrum was announced by Mrs. Fleming[5] in 1893. The following remark appears in the

[1] *Contributions from the Mount Wilson Observatory*, No. 295.

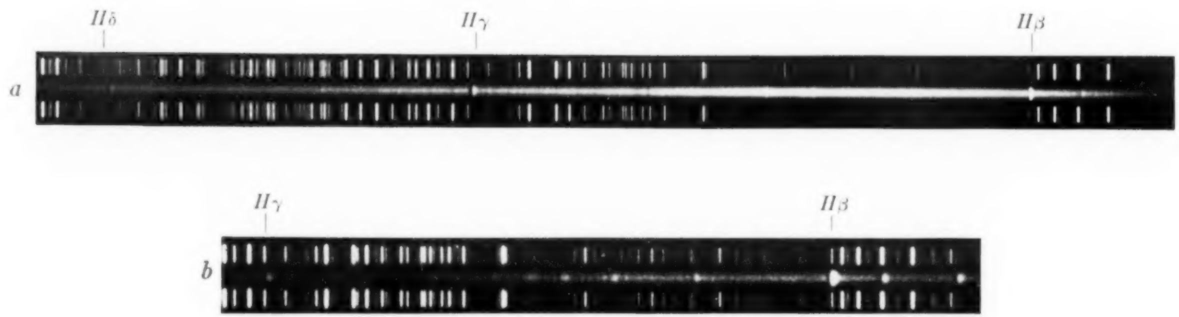
[2] Miss Maury, *Harvard Annals*, **28**, 101 (remark 165), 1897; Belopolsky, *Astrophysical Journal*, **10**, 319, 1899; Frost, *Astrophysical Journal*, **35**, 286, 1912; Merrill, *Lick Observatory Bulletins*, **6**, 156, 1911; **7**, 24, 1913.

[3] *Harvard Annals*, **76**, 31, 1916.

[4] *Publications of the Observatory, University of Michigan*, **2**, 71, 1916.

[5] *Astronomy and Astrophysics*, **12**, 810, 1893. The identification given in this reference is incorrect.

PLATE XII



a. H.D. 51480 1921 March 28
b. DM. $-27°$ 11944 1922 May 16

Henry Draper Catalogue: "The line $H\beta$ is bright, and bright lines or spaces not due to hydrogen are seen. The lines are narrow and the line K is as strong as in Class A2."

The Mount Wilson observations with slit spectrographs are listed in Table I. Several plates taken in the course of the general spectroscopic program have been available for the present investigation through the kindness of Mr. Adams.

Several lines exhibit both bright and dark components as in P Cygni, but other lines appear only in absorption (see Table II). The hydrogen lines closely resemble those of P Cygni in structure and intensity. Silicon, nitrogen, and helium lines are weaker than in

TABLE I

SPECTROGRAMS OF H.D. 51480

Date	Spectrograph	Camera	Region
1919 October 14	1-prism	18-inch	Blue-violet
1920 November 20	1-prism	18-inch	Blue-violet
1921 January 29	1-prism	18-inch	Blue-violet
1921 February 22	1-prism	18-inch	Blue-violet
1921 March 28	1-prism	18-inch	Blue-violet
1921 November 12	1-prism	18-inch	Blue-violet
1924 March 24	1-prism	18-inch	Blue-violet
1924 December 14	1-prism	18-inch	Blue-violet
1925 January 3	Grating	10-inch	Red

P Cygni, while the enhanced iron lines and λ 4481 of magnesium are stronger. The general character of the spectrum is the same on all eight plates of the blue-violet region. The only very definite changes noticeable on inspection are, in fact, slight variations in the intensity of the absorption at $H\beta$. Measurements of the principal lines observed are given in Table II. As most of the lines are unsymmetrical and rather ill defined, their displacements can be determined with only moderate accuracy. The figures in Table II are the means from two independent measures of each plate. A few indistinct lines were measured but once, and the resulting displacements are in parentheses. All the values are reduced to the sun.

The effective displacements of the various absorption lines on individual plates, as well as in the mean, differ by many kilometers. After making allowance for the large errors of measurement,

TABLE II

MEASURED DISPLACEMENTS IN THE SPECTRUM OF H.D. 51480

LINE	1919 OCT. 14		1920 NOV. 20		1921 JAN. 29		1921 FEB. 22		1921 MAR. 28		1921 NOV. 12		1924 MAR. 24		1924 DEC. 14		MEAN			P CYGNI (FROST)
	A	E	A	E	A	E	A	E	A	E	A	E	A	E	A	E	A	E	E–A	E–A
	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	Ang.	Ang.
4101.74 *Hδ*	....	....	−110	+ 51	−119	+ 83	....	....	−101	+ 49	−119	+ 44	....	....	− 70	+ 82	−104	+ 62	2.28	2.28
4178.87 *Fe*	....	....	....	....	111	....	(−66)	....	55	....	....	....	....	....	32	....	66	....	....	....
4233.14 *Fe*	....	....	94	....	107	71	70	....	86	52	87	(59)	....	....	34	97	80	71	2.13	....
4340.47 *Hγ*	−100	+ 53	116	40	109	70	113	+ 26	112	51	134	39	−96	+ 41	77	76	107	55	2.34	2.58
4351.77 *Fe*	....	....	....	....	....	....	....	....	75	72	....	....	64	....	29	77	54	74	1.84	....
4471.48 *He*	....	....	(68)	....	139	....	(67)	....	50	....	109	....	....	....	105	....	90	....	....	....
4481.22 *Mg*	....	....	62	....	125	....	72	....	50	....	124	....	43	....	48	....	75	....	....	....
4508.29 *Fe*	....	....	....	....	....	....	....	....	(72)	....	....	....	....	....	....	....	72	....	....	....
4549.47 *Fe*	....	....	....	....	70	(117)	96	....	70	....	102	....	70	....	28	127	72	123	2.96	....
4583.84 *Fe*	....	....	....	....	....	(59)	100	49	58	79	106	38	(− 75)	82	....	(110)	82	72	2.34	....
4861.33 *Hβ*	−143	+ 60	−162	+ 40	−186	+ 97	−146	+ 26	−138	+ 52	−139	+ 34	....	+ 51	−100	+ 83	−145	+ 55	3.24	3.46
Mean	−122	+ 56	−102	+ 44	−121	+ 82	−91	+ 34	− 79	+ 59	−115	+ 41	−70	+ 58	− 58	+ 92	− 94	+ 58	....	....

discrepancies remain which must be inherent in the star's light. The average negative displacements of the absorption components of the hydrogen lines, especially $H\beta$, are greater than those of the other lines, and this is probably caused by the greater strength of the emission components. *He* λ 4471 is broad and difficult to measure. The emission lines, as in P Cygni, are in better agreement than the absorption lines. Possibly their agreement is within the errors of measurement. λ 4549 seems to be outstanding, but its measured displacement has small weight as it may be partially blended with neighboring lines.

Several plates show systematic residuals from the mean. The emission lines of the plate of February 22, 1921, for example, give an average displacement 27 km/sec. less than the mean; those of the plate of December 14, 1924, 34 km/sec. more. On the latter plate the absorption lines differ from the mean in the same sense as the emission lines. Whether the shifts are periodic or irregular cannot be determined from the plates now available. In addition to bodily displacements of the spectrum, possibly due to orbital motion, certain other changes appear to exist. The third plate, for example, gives a considerably greater mean difference between absorption and emission lines than do other plates.

The displacement of the strong emission $H\alpha$ line as measured on the plate of January 3, 1925, is +72 km/sec., in approximate agreement with the mean result from the other bright lines.

DM. $-27°$11944

A very strong bright $H\alpha$ line was found in this spectrum on an objective-prism plate taken August 4, 1920, by Mr. Milton L. Humason.[1] It appears broad even with the low dispersion employed (440 A per mm), but on the best plates is sharply bounded on the short wave-length side; it is probably of the P Cygni type. It is extremely strong compared to the adjacent continuous spectrum.

The observations of the blue-violet region photographed with a single-prism spectrograph attached to the 100-inch reflector are listed in Table III.

[1] *Publications Astronomical Society of the Pacific*, **34**, 294, 1922; *Mt. Wilson Contr.*, No. 294; *Astrophysical Journal*, **61**, 389, 1925.

Numerous lines exhibit the P Cygni characteristics in a marked degree. The bright portions are very broad, and in this respect the star occupies a position intermediate between P Cygni and a typical nova. Thus DM. $-27^{\circ}11944$ supplies a missing link in the chain of bright-line objects connecting ordinary B-type stars with novae.

TABLE III

SPECTROGRAMS OF DM. $-27^{\circ}11944$

Date	Camera	Date	Camera
1921 March 30	18-inch	1921 August 13	7-inch
1921 April 10	18-inch	1922 May 16	18-inch
1921 April 29	18-inch	1923 July 27	10-inch
1921 June 21	18-inch	1924 August 20	10-inch

The bright lines are difficult to measure on account of their diffuse, unsymmetrical character, and, except for a few of the strongest, their lack of contrast with the continuous spectrum. A few of the dark lines are fairly well defined, though unsymmetrical; but most of them are weak and, as a whole, are incapable of accurate measurement. The lines (bright and dark) which can be identified are ascribed to hydrogen, helium, or enhanced iron. About all that can be said concerning the displacements is that those of the bright lines are small and positive, those of the dark lines large and negative. Because of the southern declination of the star, we have not been able to secure well-exposed spectrograms with the 18-inch camera. Even with the shorter cameras the continuous spectrum, although strong at $H\beta$, does not extend beyond $H\gamma$. Probably the real spectral intensity-curve drops more rapidly between these two lines than for a normal B-type star as the direct image has a decidedly reddish color, although this may be due in part to the very intense $H\alpha$ line.

Measurements of line-widths are given in Table IV. They are merely rough approximations as the results depend to a considerable extent upon the judgment of the observer as well as upon the effective exposures of the plates. Frost's values for P Cygni are given for comparison.

No change has appeared in this spectrum during the three-year interval covered by our observations. Moreover, the light is sensibly constant, for "if variability now exists it probably does not exceed a tenth of a magnitude in range, as no variation is found on the Harvard photographs,"[1] fifty-one in number, extending from 1890 to 1918.

TABLE IV

APPROXIMATE LINE-WIDTHS IN ANGSTROMS

LINE	−27°11944		P CYGNI (FROST)	
	A	E	A	E
4340 $H\gamma$	....	4.3	1.5	3.0
4471 *He*	....	4.3	1.6	2.2
4583 *Fe*	3.7	6.2	....	....
4712 *He*	....	5.0	....	....
4861 $H\beta$	4.8	6.8	2.0	4.3
4923 *He+Fe*	4.3	6.8	....	....
5017 *He+Fe*	6.4	7.5	....	....

MOUNT WILSON OBSERVATORY
March 1925

[1] *Harvard College Observatory Bulletin*, No. 807.

THE THEORY OF THE STELLAR ABSORPTION COEFFICIENT[1]

By S. ROSSELAND

ABSTRACT

The stellar absorption coefficient.—A reliable theory of the absorption coefficient of strongly ionized matter is a necessary complement to any theory of the internal constitution of stars. In the present paper such a theory is developed deliberately as a generalization from X-ray absorption experiments only, and partly checked by experiments of Anderson and Harrison. On the basis of this theory the expression of the flux of radiant energy in a star is calculated, and the result compared with Eddington's theory of a star in radiative equilibrium.

The theory of the opacity of an ionized gas has acquired much interest for astrophysics through Eddington's work on "The Internal Constitution of the Stars." In fact, a reliable knowledge of the absorption coefficient of gases under all conditions would enable us to draw far-reaching conclusions about the distribution of elements throughout a star and about the localization of the sources of stellar energy, questions which are fundamental for stellar theory.

The problem of determining the absorption coefficient of stellar matter can scarcely be solved by direct comparative experiments in the laboratory as, apart from the superficial layers of a star, stellar temperatures are likely to be much higher than those accessible to terrestrial experiments. The frequencies for which the absorption of radiation in the interior of a star mainly takes place, however, can scarcely be larger than those of soft X-rays, corresponding to L-radiation from elements of median atomic numbers. This fact directly suggests the application of X-ray data to the stellar problem, a question which has already been discussed by Eddington,[2] whose discussion, however, was concerned with the arithmetic mean value of the absorption coefficient, whereas theory requires something like the harmonic mean value,[3] and when the absorption coefficient varies rapidly with the frequency of the incident radiation the results given by the latter procedure may differ widely from those given by

[1] *Contributions from the Mount Wilson Observatory*, No. 296. This paper is the continuation of a paper on "Transmission of Radiation" in *Monthly Notices, R.A.S.*, **84**, 52, 1924.

[2] *Monthly Notices, R.A.S.*, **84**, 104, 1924.

[3] *Ibid.*, **84**, 525, 1924.

the former. For this reason we shall in the following reconsider the whole problem on the new basis. Actually, the final result does not differ much from that arrived at by Eddington; but this is not due to any triviality as regards the two modes of calculating the absorption coefficient. Thus, the contribution to the stellar absorption coefficient considered by Eddington was due to absorption by atoms in the normal state, while the present theory assigns an essential rôle to atoms excited to higher quantum states. Moreover, an attempt is made to distinguish clearly between what may be taken over directly from laboratory observations and what is more or less hypothetical.

In considering the problem of transmission as we meet it in the interior of a star, the absorption in a line spectrum is clearly of little significance, as this process can only bar the passage of the radiation confined in very small frequency intervals of the spectrum. It is therefore sufficient for such purposes to consider the continuous absorption setting in at the series limits, which is analogous to the continuous X-ray absorption. In the following we shall consider briefly the theory of absorption in general, and indicate a tentative generalization from results on X-ray absorption to absorption in other spectral regions. This theory will then be applied to the calculation of the stellar absorption coefficient, and the relation of the results to the theory of the radiative equilibrium of a star discussed.

1. *Formal theory of absorption.*—We shall in the following be concerned with absorption in the restricted sense that the energy of the absorbed radiation is converted into kinetic and potential energy of an electron which is free after the absorption process. For the general theory it is convenient to consider the absorbing atoms to be parts of a system in thermodynamic equilibrium, and the beam of incident radiation to correspond to black-body radiation for the temperature in question. In this special case the radiation absorbed within any infinitely small frequency interval will be counterbalanced by an equally large emission of radiation in the same interval. The absorption coefficient may thus be expressed in terms of a certain emission coefficient which is more easily submitted to theoretical calculations and interpretation.[1] As this latter, as well as the absorption coefficient itself, is essentially dependent upon purely atomic

[1] Cf. H. A. Kramers, *Philosophical Magazine*, **46**, 836, 1923.

properties, the result is in itself independent of the assumption of a thermodynamic equilibrium, and susceptible of application to quite general cases. Consider, thus, an assembly of atoms and free electrons in a state of thermodynamic equilibrium at an absolute temperature T, and let there be n_e free electrons, n_i atoms with a vacant quantum state i, and n^i atoms with the i-state filled up—all figures referring to unit volume. In considering collisions between electrons and atoms the motion of the latter will be neglected. The atoms with a complete i-state will absorb the radiation of a frequency ν under expulsion of electrons with a velocity v according to the $h\nu$-relation,

$$h\nu = \tfrac{1}{2}mv^2 - E_i \,, \tag{1}$$

where $-E_i$ is the energy which must be given to the electron in the i-state in order to transfer it to an infinite distance from the atom. If $B_\nu d\nu$ is the energy density of temperature radiation in the frequency interval ν to $\nu+d\nu$, the energy absorbed by the atoms in the i-state in the given frequency interval per unit volume in unit time is

$$n^i x_\nu^i c B_\nu d\nu \,, \tag{2}$$

where x_ν^i is the absorption coefficient and c the velocity of light. These absorption processes must be counterbalanced by an equal number of emission processes in which free electrons with velocity v collide with atoms with initially vacant i-states and are bound in these states in the collision. Consider collisions in which the distance from the atomic nuclei to the direction of the initial velocity of the colliding electrons is confined in the interval p to $p+dp$, the number of which is

$$n_i dn_e v 2\pi p dp \tag{3}$$

per second, where dn_e is the number of free electrons per unit volume in the velocity interval v to $v+dv$. Denoting by P_i the probability that an electron shall be bound in an i-state in the collision process, we find the total emission of radiation in the frequency interval in question by multiplying expressions (3) by $h\nu P_i$ and integrating over all possible values of p. The result thus obtained must be equal to the expression (2), or

$$n^i x_\nu^i c B_\nu d\nu = 2\pi v h\nu n_i dn_e \int_0^\infty P_i p dp \,. \tag{4}$$

The function P_i must, according to the well-known theory of Einstein,[1] be independent of the energy density of radiation to the same degree of approximation that B_ν is reproduced by Wien's expression

$$B_\nu \backsim \frac{8\pi h\nu^3}{c^3} e^{-\frac{h\nu}{kT}} ,$$

and we may, therefore, without in the least committing ourselves to approximate results, assume the temperature to be so low that such conditions will obtain. Introducing the ordinary expression for $n_i dn_e / n^i d\nu$ from statistical theory

$$\frac{n_i}{n^i}\frac{dn_e}{d\nu} = \frac{4\pi h m^2}{\sigma_i} v e^{-\frac{h\nu}{kT}} , \tag{5}$$

we find, finally,

$$x_\nu^i = \frac{h m^2 c^2 v^2}{2\sigma_i \nu^2} 2\pi \int_0^\infty P_i p dp , \tag{6}$$

where σ_i is the a priori probability of the atom with the i-state filled, relative to the state with the i-electron missing. The quantity

$$y_\nu^i = 2\pi \int_0^\infty P_i p dp , \tag{7}$$

just as x_ν^i, has the dimensions of an area, and may be called the effective cross-section of the atom for capturing electrons of velocity v to the i-state. Conversely, when x_ν^i and y_ν^i are known, we may, by retracing the steps, obtain the value of $n_i dn_e / n^i$, and we are thus able to calculate the degree of ionization of the substance without going through all the steps of the ordinary statistical theory. This process, suitably generalized, may be susceptible of application to the problem of calculating the ionization of the material of gaseous nebulae where ordinary thermodynamic formulae very probably would break down.

For the further elucidation of the function P_i we may consider the problem from the point of view of the correspondence principle.[2]

[1] *Loc. cit.*

[2] Cf. H. A. Kramers, *loc. cit.*, where the discussion is carried out at length. The discussion here has a different purpose, as we wish only to obtain a method of generalizing experimental results.

Consider thus the general problem of a collision between an electron and an atomic system on the basis of ordinary electrodynamics. The total loss of energy of the electron during the collision will be given by an expression of the form

$$\delta E=\frac{2e^2}{3c^3}\int\limits_{-\infty}^{\infty}[1-\beta^2(\lambda-1)(3\lambda-1)]\,\frac{F^2}{(1-\beta^2)^3}\,dt\ ;\qquad \beta=v/c\ . \tag{8}$$

where F is the acceleration and v the velocity of the electron, while arc cos λ is the angle between these two vectors. Assume this energy to be resolved into a continuous spectrum such that $E_w dw$ is the energy falling in the frequency interval dw. For velocities of the electron which are small compared to the velocity of light, the mean value of the acceleration F at a distance r from the nucleus may be expected to have the form

$$F=\frac{N_f e^2}{mr^2}\ , \tag{9}$$

where $N_f e$ is the net electric charge contained within a sphere of radius r and having the nucleus for a center. As the emission very probably is principally stimulated during the passage close to the nucleus, we may expect that the expression of E_w will contain $(Ne^2)^2$ as an essential factor, where N is the atomic number. Denoting the initial value of the angular momentum of the electron round the nucleus by M it is possible to write

$$E_w=\tfrac{2}{3}\frac{e^2}{c^3}\left(\frac{Ne^2}{M}\right)^2 G_w\ ,\qquad M=mvp \tag{10}$$

where G_w is a zero dimensional quantity which in general may be expected to be a continuous and smooth function of p, v, and w. There is no doubt that in an actual collision in nature the frequency distribution of energy in the emitted radiation will be essentially different from that predicted by a formula such as (10). In particular, it is known with certainty that the spectrum actually emitted in a collision of the foregoing type cannot extend materially toward larger frequencies than the limit $\nu=\Delta E/h$, which by the frequency relation corresponds to the total loss of energy ΔE in the collision. Moreover, the spectrum of the radiation emitted by a large number

of electrons suffering collisions of identically the same type may be expected to consist of a typical line spectrum, one for each possible quantum state of the electron when bound in the atom, and a continuous spectrum corresponding to the fact that some electrons will radiate without being bound. We must, therefore, if we expect classical electrodynamics to be applicable to the case at all, postulate a general *correspondence* between the classical intensity of radiation in the interval dw and the intensity of radiation really emitted in the interval $d\nu$, such that each dw contributes an amount of energy

$$\gamma_i(\nu, w)E_w\, dw\, d\nu \tag{11}$$

to the frequency interval $d\nu$ in question. As the total amount of energy absorbed when an electron is shot out of the atom under absorption of radiation of frequency ν is $h\nu$, we have by (4) the expression

$$\frac{1}{h\nu}\int_0^\infty \gamma_i(v, w)E_w\, dw \tag{12}$$

which is by definition equal to the function which was previously denoted by P_i. By combination of (6), (10), and (12) we obtain

$$x_\nu^i = \frac{2\pi\, e^6\, N^2}{3c\, \sigma_i\, \nu^3}\int_0^\infty \int_0^\infty \gamma_i(\nu, w)G_w\, dw\, dp/p\ . \tag{13}$$

The function $\gamma_i(\nu, w)$ can have appreciable values only when w is in the vicinity of ν, otherwise the postulate of a correspondence between classical and quantum radiation would not have much sense. The integral

$$\int_0^\infty \gamma_i(\nu, w)dw \tag{14}$$

defines a frequency interval which must be roughly proportional to the energy difference between the i-state of the atom and an adjacent state, in order to comply with the requirements of the frequency relation. This energy difference is in turn approximately proportional to the square of the effective nuclear charge and inversely proportional to the third power of the effective principal quantum number n of

the electronic group in question. Thus[1]

$$\int_0^\infty \gamma_i(\nu, w)G_w dw \backsim G_\nu \frac{\partial E_i}{\partial nh} \propto \frac{N^2}{n^3} G_\nu \tag{15}$$

or finally

$$x_\nu^i = \frac{\nu^{-3}N^4 r}{\sigma_i n_i^3} K(\nu) \; ; \qquad K(\nu) = \text{const.} \int_0^\infty G_\nu \frac{dp}{P} \, , \tag{16}$$

where r is the number of electrons in the group in question.

One might for a moment be inclined to the belief that it would be possible to proceed in the reverse direction and from measurements of the absorption coefficient learn something about the motion of electrons in collision with atoms. The information which can be obtained in this way is, however, not very definite as it is only involved through the function G_ν, which, unfortunately, remains undetermined to a rather wide extent. In fact, the integral equation for G_ν given by theory

$$\int_0^\infty G_\nu \frac{dp}{P} = CK(\nu) \, , \tag{17}$$

where C is some constant, has the solution

$$G_\nu = \frac{CK(\nu)Y(pU)}{\int_0^\infty Y(x)dx/x} \, , \tag{18}$$

where $Y(x)$ is an *arbitrary* function of one argument only, and U an arbitrary function of the frequency ν. This result contains, in fact, practically all that continuous absorption experiments can teach us about the motion of electrons in collision with atoms.

2. *Generalization of the X-ray absorption coefficients.*—According to experimental results,[2] the absorption coefficient for monochromatic X-rays absorbed in a definite electronic level i of the atom is of the form

$$x_\nu^i = \nu^{-3}N^4 A_i \, , \tag{19}$$

[1] This is the hypothesis which was adopted by Kramers, *loc. cit.*

[2] Cf. F. K. Richtmeyer and F. W. Warburton, *Physical Review*, **22**, 539, 1923.

where A_i is constant as long as the frequency in question is larger than $h^{-1}E_i$, E_i being the energy necessary to remove an electron from the level in question, and zero for smaller frequencies. The validity of this formula, especially for absorption of hard radiation in the K-level, has been tested for a large number of elements ranging from $N=6$ (carbon) to $N=82$ (lead). Comparing (19) with (16) we see immediately that this result demands that the function $K(\nu)$ be independent of ν for absorption in the levels in question. Experiments show further that the constant A_i decreases rapidly when passing from the K to the L-level, and so on. This fact is naturally attributed to the changes in the factor $r/n^3\sigma_i$, which are of the right order of magnitude. It would therefore seem that the function K is an absolute constant for all levels in question.

The experiments at hand are only concerned with the case of electronic groups which are located so close to the nucleus that the effective atomic number of the group is practically equal to the atomic number of the atom. When we proceed to consider absorption in electronic groups which are located so far from the nucleus that the screening effect of the intervening electrons on the motion of the electrons in the group becomes appreciable, we have no reason for expecting formula (19) to be applicable. In default of experimental evidence it may be of interest to consider the alteration in this formula which would be suggested by the formal theory given above. According to theory, the factor N^4 is a product of two factors, each of which proceeds from quite different sources. Thus one factor N^2 was introduced by the fact that a free electron colliding with an atom emits the more radiation the larger the nuclear charge of the atom. That this process alone will introduce a factor N^2 is independently evidenced by the fact that the total emission of general x-radiation from a target bombarded with electrons of given velocity is proved by experiments to be proportional to the square of the atomic number of the substance constituting the target. Therefore, on considering absorption of radiation in outer atomic groups, the foregoing factor N^2 would be expected to be carried over with little alteration, or at most with the introduction of a small screening constant a_i, thus giving $(N-a_i)^2$ for N^2 in the formula. The second factor has been shown to measure the energy difference be-

tween successive quantum states. From X-ray spectroscopy this factor is known to vary considerably from level to level, and this variation we shall account for by a second screening constant, β_i. The final form of formula (16) is thus

$$x_\nu^i = \nu^{-3}(N-\alpha_i)^2(N-\beta_i)^2 \frac{K(\nu)r}{\sigma_i n_i^3} \,. \tag{20}$$

Measuring all quantities relative to the corresponding quantities in the K-level, we find the numerical values of K, according to the results of Richtmeyer, to be

$$K = 2.2 \cdot 10^{-2} c^3 \text{ c.g.s.} \,,$$

where c is the velocity of light. The foregoing formula refers to a definite level. In general, a number of levels will come into play, and, in order to obtain the mean absorption coefficient averaged over all atoms present in the system, each expression such as (20) must be multiplied by the relative probability τ that the atom shall be found in this particular level, and the sum taken over all levels which can come into consideration.

It is of particular interest to consider the case in which the distribution of atoms over the various quantum states corresponds to a state of temperature equilibrium. The numbers τ are then given by the expression

$$\tau_i = \frac{\sigma_i e^{-\frac{E_i}{kT}}}{f(T)} \,; \qquad f(T) = \sum_s \sigma_s e^{-\frac{E_s}{kT}} \,, \tag{21}$$

and the mean absorption coefficient, consequently, by

$$x_\nu = \sum_k \frac{\tau_k K r \nu^{-3}}{n_k^3 \sigma_k} (N-\alpha_k)^2 (N-\beta_k)^2 \,, \tag{22}$$

where due care must be exercised to include only those states which by the frequency relation can come into play in the absorption process. It is here assumed that the quantum states of the atom in question are sharply defined, otherwise the foregoing series of discrete terms would have to be replaced by an integral over the infinite number of possible states, and conditions would approach those contemplated in the classical statistical theory. The foregoing theory is also susceptible of application to this case, in particular, when the

broadening of the states is so advanced that the distribution of atomic states in the phase space may be considered to be continuous. For this limiting case we may effect the summation involved in expression (22) by directly referring to equation (4) from which we started. In fact, integrating both sides of this expression over all possible kinds of atoms (left side) and all possible velocities of the electrons (right-hand side), we find that the resulting expression may be written in the form

$$n^{\circ}x_{\nu}cB_{\nu}=2\pi\, h\nu \sum_{a} n_a \int v\,dn_e \int_{\circ}^{\infty} P_i\, p\,dp \;, \tag{23}$$

where n° is the total number of atoms per unit volume, x_{ν} the mean value of the atomic absorption coefficient, and n_a the number of atoms in a particular state of ionization, the summation being extended to all kinds of atoms. Denote the energy emitted when a free electron with zero velocity falls to the lowest possible quantum state of the atom by $h\nu_a$, and by ν_a', the largest frequency of the classical spectrum which corresponds to ν_a. The integration as regards v in the foregoing expression is then to be extended from the lower limit v_0 given by

$$\tfrac{1}{2}mv_0^2=h(\nu-\nu_a') \tag{24}$$

to infinity, provided $\nu>\nu_a'$, and from $v=\circ$ for the case that $\nu \lesseqgtr \nu_a'$. Introducing the proper expressions for P_i and dn_e/dv, and remembering that

$$G_{\circ}=\int_{\circ}^{\infty} G_{\nu}\,dp/p \tag{25}$$

is independent of v, we obtain finally by integration, the expression for x_{ν},

$$x_{\nu}=\tfrac{4}{3}\frac{e^6}{c^4}G_{\circ}\left(\frac{2\pi}{m^3kT}\right)^{\frac{1}{2}} n_e B^{-1}\nu \sum_{a}\frac{n_a}{n^{\circ}}(N-a_a)^2 e^{-h\frac{(\nu-\nu_a')}{kT}} \;; \qquad n^{\circ}=\sum_{a} n_a \;. \tag{26}$$

It is to be understood that an exponential factor is to be taken equal to unity in the foregoing formula in case $\nu \leqq \nu_a'$. The constant G_0 in the foregoing expression may be determined from X-ray data with some accuracy. In fact, the detailed expression for K suggested by theory is, from (15) and (16), seen to be

$$K=\frac{(2\pi)^3 e^{10} m}{3ch^6}G_{\circ} \tag{27}$$

Here the a priori probability is measured in units of h^3 while E_i, by atomic theory, is assumed to be given by

$$E_i = -\frac{2\pi^2 e^4 m N^2}{h^2 n_i^2} . \tag{28}$$

From the experimental value of K and from (27) it follows that

$$G_0 = 34.7 . \tag{29}$$

There can be little doubt that for large values of the quantum numbers the law suggested by (15) must be rather accurate, but for the case of the K-level, when the principal quantum number is unity, this hypothesis may be subject to some uncertainty, but probably not by more than a factor $2^{\pm 1}$. By observations of X-ray absorption in higher levels this uncertainty regarding the value of G_0 might be much reduced.

As it stands, formula (26) gives the aggregate absorption coefficient as it is caused by atoms, or possibly free electrons, which are in collision with the atoms. Putting all ν'_a equal to zero gives the absorption by free electrons alone:

$$x_\nu^e = \frac{4}{3} \frac{e^6}{c^4} \left(\frac{2\pi}{m^3 kT} \right)^{\frac{1}{2}} G_0 B_\nu^{-1} n_e e^{-\frac{h\nu}{kT}} \sum_a \frac{n_a}{n^0} (N - a_a)^2 . \tag{30}$$

This formula corresponds in a certain sense to that derived in the classical electron theory for the absorptive power of metals, but with the difference that it is not restricted to long wave-lengths, as was generally the case in the classical theory.[1] As one might have anticipated on general grounds, formula (30) predicts a strong increase in the absorptive power with increasing wave-lengths of the incident radiation, while for short wave-lengths it drops very rapidly to an insignificant value. To the absorptive power of the electrons due to collision with atoms,[2] must be added the scattering power of the

[1] In fact, formula (30) is not applicable to absorption of radiation of very long wave-length, which is directly evidenced by the fact that when $\nu \rightarrow 0$ it would predict an infinitely large absorption, while a revised theory would lead to an asymptotical equality of x_ν to $4\pi\sigma$, where σ is the electric conductivity of the substance, which must have a finite value.

[2] The effect of collisions between the electrons themselves will be in the sense of diminished absorption, as the electrical moment of two colliding electrons is practically zero, due to equality of masses and charges. The diverging result obtained by Eddington (*Monthly Notices, R.S.A.*, **84**, 104, 1294) arises from the fact that he neglects the systematic relation between the accelerations of two electrons in collision.

electrons themselves when they remain perfectly free. This scattering coefficient is given by the well-known formula of J. J. Thomson

$$x=\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \tag{31}$$

per electron for all wave-lengths longer than those of hard X-rays.

The foregoing generalization of the results from X-ray experiments is purely tentative and intended to serve only as a guide for applications to stellar theory. For this purpose it is very important to have the formulae checked in some way, in order to gain an idea of the directions in which deviations from theory are likely to take place. Unfortunately, the experimental evidence on this point is very scanty, as the only check at hand is afforded by a few absorption measurements in the optical region where we should hardly expect an extrapolation from X-ray experiments to afford any approximation at all.

Harrison[1] appears to be the first to have made quantitative measurements on the continuous absorption setting in at the series limit of the principal series of gases and vapors. He measured the absorption coefficient in the region of continuous absorption beyond the series limit of the principal series of sodium, but states that the measurements are rather uncertain, although they undoubtedly serve as a first orientation in the field. The necessary data kindly put at my disposal by Dr. Harrison himself show that the absorption coefficient per atom for a definite small frequency interval just beyond the series limit ($\lambda = 2414$ A) is of the order of 10^{-19}, and that the absorbing power decreases rapidly when proceeding to shorter wave-lengths. The latter circumstance is in fair-enough agreement with expectations, but the absolute value of the coefficient appears to be much smaller than can be accounted for on the basis of (22) or (19), however we twist them. In fact, even by putting $(N-\alpha_i)\times(N-\beta)=1$, which certainly is the smallest value that can possibly come into consideration, and by taking $n=1.5$ as claimed by spectroscopy, we obtain $x=7.10_{-}^{17}$, which is several hundred times larger than the experimental value.

[1] *Physical Review*, **24**, 466, 1924.

Anderson[1] has investigated the opacity of electrically exploded wires. During the explosion the wires evaporate and the vapors are raised to a temperature of about 20,000° K., as a consequence of which we no longer deal with atoms in the neutral state only, as is the case in Harrison's experiments, but with atoms distributed over a variety of quantum states and partly ionized. Consequently, we should expect on theory that the absorption spectrum of the vapors would be a superposition of a variety of series-limit absorption spectra, which would be strongest in the short wave-length region where we are concerned with absorption by atoms in the lowest possible quantum state, and, because of absorption by atoms excited to higher states, would extend with diminishing intensity right up to the optical region. This, in fact, appears to be in general accordance with Anderson's results. Stewart[2] suggested that the strong absorption observed in these experiments might be due to the action of free electrons. This theory, however, does not seem calculated to account for the observed facts. On any theory (as, for instance, Stewart's own, or that given above) the effect of free electrons must be confined to long wave-lengths where the time spent by an electron on a free path is small compared to the period of a light-vibration, and for short wave-lengths it must degenerate into an ordinary Thomson scattering, which is much too small to account for the absorption in question. It would, however, be interesting to have these experiments extended to the long wave-length region where the absorption by free electrons necessarily must make itself felt. An experimental confirmation of this expectation would be all the more interesting as it would afford a case where the classical electron theory is applicable in principle.

Anderson's experiments appear thus to be in qualitative agreement with the theoretical considerations given above. Moreover, they appear also quantitatively to conform much closer to theory than was the case for Harrison's experiments. Dr. Anderson has kindly informed me that the absorption coefficient per atom of exploding iron vapor (temperature and pressure estimated as 20,000° K. and 4 atm., respectively) appears to be of the order of $2 \cdot 10^{-19}$ for

[1] Cf. *Proceedings of the National Academy of Sciences*, **8**, 231, 1922.

[2] *Nature*, **111**, 186, 1923.

the spectral region λ 4000 A. This absorption must be due to atoms excited in higher quantum states, as the absorption by atoms in the normal state, whether neutral or ionized, must be confined to the ultra-violet region. The case is therefore one which suggests the application of formula (26), which just corresponds to the overlapping absorption by excited atoms. There is apparently only one stage of ionization in play in the experiments, and we may therefore write (26) in the form

$$x_{4000}=\frac{n_e}{n^0}\left\{\frac{n_a}{n^0}(N-a_1)^2+\left(1-\frac{n_e}{n^0}\right)(N-a_2)^2\right\}10^{-20}\ , \tag{32}$$

where we have introduced experimental data given above. The ratio n_e/n^0 is of the order $1/2$. The atomic number of iron is 26, and it is seen that it is necessary to introduce rather large screening constants in order to make the result coincide with experiment; but there is no indication of huge discrepancies, as in the case of Harrison's experiments. The clearing up of this puzzle seems rather to be the task of experimenters. Provisionally we must be content with the meager result that the extrapolation of the result afforded by X-ray experiments on optical absorption appears to indicate too strong an absorption; but even this result may be of some help for stellar theory, as will be apparent from the following discussion.

3. *The stellar absorption coefficient.*—It is easily proved[1] that in the interior of a star the net flux of radiation in the frequency interval ν to $\nu+d\nu$, through unit area, in an outward radial direction is to a high degree of approximation given by the expression

$$F_\nu d\nu=-\frac{c}{3x_\nu\rho}\frac{\partial A_\nu}{\partial r}d\nu\ , \tag{33}$$

where $A_\nu d\nu$ is the energy density of radiation in the frequency interval $d\nu$, ρ the density, and x_ν the mass absorption coefficient of the substance in question. This result rests on the assumption that the medium is at rest—thus excluding convection—the state stationary in time, and that the flux of radiation is very nearly the same for all directions. The net total flux of radiation in an outward direction is

$$F=\int_0^\infty F_\nu d\nu\ , \tag{34}$$

[1] Cf. *Monthly Notices, R.A.S.*, **84**, 525, 1924.

or, introducing the auxiliary variable x defined by

$$\frac{1}{x}\frac{\partial A}{\partial r}=\int_0^\infty \frac{\partial A_\nu}{\partial r}\frac{d\nu}{x_\nu}\,, \tag{35}$$

we obtain

$$F=-\frac{c}{3x\rho}\frac{\partial A}{\partial r}\,, \tag{36}$$

where A is the total energy density of radiation. For the case of the interior of a star the deviations from thermodynamic equilibrium must be exceedingly small, and we may replace A_ν by Planck's formula.

The auxiliary quantity x is the *stellar* absorption coefficient entering Eddington's theory of radiative equilibrium of a star, and it will be the characteristic function of the medium as regards its power of transmitting radiation *en bloc*. From (35) it is seen that x may be regarded as a kind of harmonic mean value of the absorption coefficient x_ν, and because of this fact pronounced selective absorption will be of practically no importance for the total transmission of radiation.

It is not difficult to derive the general expression for x on the basis of the theory of the absorption coefficient as tentatively given above. The result, however, is very complicated, and, as there is no hope of any refined test, we shall limit the consideration to the derivation of an upper limit to x, which will also bring out its general dependence upon temperature and density of the medium. For this purpose we assume the quantum states to be so strongly broadened that the absorption coefficient may be considered as a continuous function of the frequency. This will certainly be the case as soon as we consider the long wave-length region of the spectrum (relative to the wave-length of maximum energy) where the absorption is smallest and the absorption limits in all cases would follow one another so closely as to form an almost continuous sequence. Under these circumstances the absorption coefficient is given by (26). For the transmission of radiation *en bloc* it does not matter much what expression we use for the absorption coefficient in the very long and very short wave-length region, since the absorption there is in any case so large

as to cut out the essential contributions to the integral in (35). We may thus drop the exponential factors in expression (26) altogether, proceeding formally as in a *purely classical* theory of absorption. Proceeding in the manner suggested above, we obtain for the stellar absorption coefficient per atom, if we neglect the small scattering of free electrons not in collision,

$$x=\frac{n_e}{T^{\frac{7}{2}}}D\frac{e^6}{c}\frac{h^2G_0}{\sqrt{m^3k^7}}\sum_a\frac{n_a}{n^0}(N-a_a)^2\,;\qquad D=\sqrt{\frac{2}{\pi}}\frac{4\,!\,2^7}{7\,!\,3}\frac{\sum_1^\infty n^{-4}}{\sum_1^\infty(n-1)\left(\frac{2}{n}\right)^7}=0.16\,. \tag{37}$$

Here n_e is given by the approximate formula,

$$n_e=\frac{\rho}{H}\left(\frac{Z_a}{A_a}\right), \tag{38}$$

where H is the mass of a hydrogen atom, and Z_a and A_a are the numbers of positive charges and the atomic weight of any of the atoms present, the ratio being assumed to be the same for all atoms. The final expression of x referred to absorption per unit mass is

$$x=\frac{\rho}{T^{\frac{7}{2}}}\cdot\frac{Z_a}{A_a}\,\frac{De^6h^2G_0}{c\sqrt{m^3k^7}\sum_a\frac{n_a}{n^0}A_a}\sum_a\frac{n_a}{n^0}(N-a_a)^2\,. \tag{39}$$

This expression shows the same dependence upon temperature, density, and atomic number as the different expressions derived by Eddington, and numerically there will not be much difference either.

It is a little difficult to know exactly to what level in the stellar interior the calculation of the stellar absorption coefficient refers in Eddington's theory of radiative equilibrium, as the value entering into the mass-luminosity relation is some kind of average to be taken over a region where the mean molecular weight of stellar matter can be assumed to be constant. Since, however, the temperature does not vary much within a star on the theory in question, which, moreover, entails the constancy of ρ/T^3, the absorption coefficient will vary very slowly for the part of the star where the mean molecular weight can be considered as constant to any degree of approximation. It would therefore seem justifiable to refer the calculations to

the center of the star, where the theory for the case of Capella[1] would give the following values:

$$T=9.56\cdot 10^{6}\,{}^{\circ}K.\,;\qquad \rho=0.1474\ gcm^{-3}\ ,$$

the mean molecular weight being assumed to be equal to 2.2. Introducing these values in (39), we obtain

$$x=2.4\,\frac{Z_a}{A_a}\,\frac{\sum_a \frac{n_a}{n^0}\,(N-a_a)^2}{\sum_a \frac{n_a}{n^0}\,A_a}\,cm^2\,g^{-1}\ .$$

The maximum possible value of this expression would be obtained if the star consisted of uranium only,[2] for which case the variable factor above is certainly less than 15; but for smaller atomic numbers it would be correspondingly less. The maximum possible theoretical value of x is thus 37, while the value estimated by Eddington from the mass-luminosity relation was $x=43$. These quantities are of the same order of magnitude. Taking as a sample of stellar matter an element of median atomic number, say iron ($N=26$), the theoretical value of the absorption coefficient would come out as about 10.[3] On the whole, it seems that the theory can conform with the requirements of Eddington's mass-luminosity relation only by assuming the core of a star to consist of elements of very large atomic numbers to a degree which surpasses what might be expected from the distribution of elements in the earth's crust or in meteors.

It seems to the writer that the essential weakness of the theory given above rests with the determination of the constant G_0, which perhaps may be 50 per cent off. Apart from that, effort has been made everywhere to obtain the maximum value of the stellar absorption coefficient, and when the result, in spite of precautions,

[1] *Monthly Notices, R.A.S.*, **84**, 104, 1924.

[2] Atomic theory seems to afford strong arguments against the possible stability of atoms essentially heavier than the uranium atom, as was pointed out by the writer in a letter to *Nature*, **111**, 357, 1923. Cf., also, N. Bohr. *Annalen der Physik*, **71**, 266, 1923.

[3] On comparing this figure with that given by Eddington's discussion of this problem it must be remembered that we use a partly empirical value of G_0, while Eddington works throughout with Kramers' theory. This latter, however, gives a value of G_0 which is about one-half that used in the text.

comes out decidedly lower than that given by theory of radiative equilibrium it seems difficult to blame the former theory. It may therefore be of interest briefly to consider different possibilities of reconciliation between the two differing results, although a final decision will not be attempted.

One might for a moment think that the fact was to be explained by assuming the core of a star really to consist of very heavy elements, sorted out and concentrated at the center by the star's gravitational field. The fact that rather heavy elements are observed at the stellar surface might seem to argue in favor of this view. The arguments, however, are not conclusive, for in the interior of a star the statistical distribution of electric charges must be such as by necessity to set up an electric field which will repel positive charges, and by the fact that positive charges on an atom under those conditions must be nearly proportional to the atomic mass, the electric repulsion will be such as largely to counteract the increased attraction on heavy atoms.[1] Hydrogen forms an exception and under certain conditions will even possess a tendency to be *repelled* from the stellar core.[2] An estimate would seem to suggest that this inner core of partial mixture would extend say halfway out from the star's center. From the boundary of this core and to the stellar surface conditions are prevalent for which at present we have no detailed theory. If this outer region of a star could be shown to be in stable equilibrium as regards vertical displacements, there might be reason for expecting an appreciable concentration of heavy elements toward the core of the star. If, on the other hand, this layer is in convective equilibrium, there would be less reason for believing in such a conclusion.

Curiously enough, reconciliation would also be brought about by assuming the stars to contain an excessive amount of the *lightest* of all elements, hydrogen, as pointed out by Eddington.[3] This result is due to the fact that the mean molecular weight of ionized hydrogen is only one-half, and the absorption coefficient calculated from the mass luminosity relation would for this case be enormously dimin-

[1] Cf. *Monthly Notices, R.A.S.*, **84**, 720, 1924.

[2] A closer discussion of this question will be given on a later occasion.

[3] *Monthly Notices, R.A.S.*, **84**, 108, 1924.

ished. There are several facts indicating that hydrogen really is present to an abnormal amount in stellar *atmospheres*,[1] but whether it is permissible to allow a sufficient amount of hydrogen to be imprisoned in the core of the star to bring about agreement with theory must provisionally remain an open question, which the calculations as regards the electrical state of the star would tend to answer in the negative.[2]

We must finally face the possibility that the assumptions on which the theory of radiative equilibrium, in the special form preferred by Eddington, are based, may require revision. The empirical evidence in favor of this theory is mainly afforded by data on the masses and luminosities of the stars, which so far agree remarkably well with the mass-luminosity relation, which is the principal outcome of the theory.[3] Analytically, however, this result cannot be taken as unambiguous confirmation of the special assumptions on which the distribution and density in the interior of the star are based, as there are an infinite number of different possible ways of distributing sources of energy within a star so as to bring about the *same* relation between luminosity and total mass, while the distribution of density and temperature is different for each case. The freedom of choice may, however, be somewhat restricted by requirements of stability, although such considerations will scarcely be sufficient for a unique solution. A reliable theory of the absorption coefficient, on the other hand, will afford a new restriction which may be sufficient to give important information about the localization of the sources of stellar energy. This question, however, will not be discussed in this paper.

MOUNT WILSON OBSERVATORY
February 1925

[1] Cf. Compton and Russell, *Nature*, **114**, 86, 1924. The suggestion advanced by these authors that the result might be a spurious effect due to metastable properties of the 2_1-state of the hydrogen atom seems to be negatived by (unpublished) calculations of the writer, showing that the maximum concentration generally to be expected will correspond to temperature equilibrium at the effective temperature of the star.

[2] Eddington has recently (*Observatory*, March, 1925, p. 73) shown that rotation of a star must set up convective currents which may be effective in intermixing different elements—at least within the surface region. (Added in proof).

[3] *Monthly Notices, R.A.S.*, **84**, 311 (plate), 1924.

REVIEWS

The Physics of the Developed Photographic Image. By F. E. Ross. New York: D. Van Nostrand Co.; Rochester: Eastman Kodak Co., 1925. 8vo. Pp. 217. 82 figures. $2.50.

A welcome addition to the library of the research worker who uses photographic methods is to be found in this book by Dr. Ross, which is Number 5 in the series of monographs on the "Theory of Photography" from the Research Laboratory of the Eastman Kodak Company. The subject matter of this monograph will especially appeal to physicists and astronomers as typical of those scientists whose quantitative work is becoming increasingly dependent upon photographic processes and the interpretations of image positions and intensities.

The exactness of the title so much needed in these days of rapidly increasing technical literature precludes any disappointment as to what may and may not be expected to comprise its contents. The book makes no pretense at being a complete treatise on the physics of photography, and can therefore omit many topics which would concern the so-called professional photographer and the photographic arts. To quote from the Preface: "It is the aim of the present volume to present data on and concepts and interpretations of the behavior and properties of the photographic emulsion which are of interest primarily to astronomers and physicists." While the book comprises Dr. Ross's own contributions, some of which have been published in this *Journal* and elsewhere, the author outlines the investigations and conclusions of other investigators in the field so that the "reader's concept of the subject as a whole may be connected and logical."

The chapter headings are as follows: chapter i, "The Developed Silver Grain"; chapter ii, "Graininess"; chapter iii, "Astronomical Photographic Photometry"; chapter iv, "Sharpness and Resolving Power"; chapter v, "Mutual Action and Adjacent Images"; chapter vi, "Film Distortion and Accuracy of Photographic Registration."

The opening chapters deal with fundamentals, discussing the developed silver grain, and detail the results of extended studies of "graininess" made chiefly in the laboratory of the Eastman Kodak Company. It

appears to be of no small practical importance that the degree of "graininess" is not seriously affected by the character of the developer used. The fact that neither the size nor position (*locus*) of the developed grain corresponds exactly with the size and position of the light-impression leads to a discussion of the "germ" of development, which is found always lying outside the exposed silver particles and forming the nucleus of the developed image. The direction of image growth, moreover, appears quite independent of the place where the light acts. Such laboratory results are of vital consequence to astrometry and other problems of photographic mensuration.

After the discussion of these problems the author devotes some seventy pages to the subject of astronomical photographic photometry, the treatment of which he divides for convenience into two subdivisions: (*a*) the increase of density with increasing exposure, and (*b*) the increase in the size of image with increasing exposure. The greater range of sensitivity in the second instance points to the advantages of focal measures as compared to extra-focal measures in astronomical problems. The question of variation in sensitivity of the emulsion over a single plate receives attention, and different methods of measurement involving both density and diameter of image are discussed at some length.

A study of the mathematical relations involved in photographic photometry is properly introduced into these chapters and typical curves exhibited for important characteristics of the plate. In consideration of the somewhat chaotic state of optical terminology which made necessary the recent canvas of the Optical Society of America, it might have proved a welcome feature had Dr. Ross introduced at this point or elsewhere in the volume a convenient summary of definitions of all fundamental quantities involved in photographic and photometric discussion. Such a summary, which might perhaps have covered a page in the Appendix, would have been of much value for reference to students of allied subjects.

It is a great convenience to find gathered together into one place the various formulae connecting density with exposure. The discussion of the reciprocity law shows that its failure in astronomical problems is due to a variety of factors and emphasizes the need of standardization in all experiments to test its validity. The somewhat unsatisfactory state of the data regarding the failure of this law leads to a postponement of a fuller treatment of the subject.

In the discussion of heterochromatic photometry, the author sounds a much-needed note of caution when he says "in so far as the physics of the heavenly bodies depends upon photometric measures and scales, ex-

treme circumspection is necessary in order that correction conclusions may be reached." He then particularly points out that on account of the relatively low penetration of feeble short wave-lengths, faint stars under long exposure will invariably appear redder than they actually are.

A discussion of image growth and turbidity leads to the treatment of sharpness and resolving power in chapter iv. Defining resolving power as the number of lines per mm just separated on the plate, the author states that the resolving power of the average fast plate is 33, whereas for the average fine grain plate 55 is given. It is of further interest to note that with red light resolving power in certain instances has been doubled. On the other hand, much inconsistency in results still obtains from different investigators.

Of particular interest to astronomers is Dr. Ross's reply to the insistent demand of celestial photographers for a high speed plate of fine grain. He points out that the best *linear* resolution of double stars obtainable with large telescopes under the most favorable conditions is about 60 μ, whereas the linear resolution obtained in the laboratory for emulsion of coarse grain and high speed is 20 μ. He therefore concludes that the true resolving power of the plate is far ahead of that which can be obtained in practice with large telescopes. Progress in the future then lies in high speed plates which, although not of finer grain, will actually effect a better resolving power by reason of the shortened exposure and consequent decrease in the effect of atmospheric tremors.

The last two chapters deal with the interaction of photographic images and film distortion. The author's extensive researches in this field make possible a very complete and authoritative statement of the present knowledge of this problem in photographic physics. The effective displacement in the case of spectral emission lines appears dependent upon a variety of causes, comprising the turbidity, Eberhard, and Kostinsky effects, each of which is discussed in turn. The mutual action of adjacent images appears more or less dependent upon the developer used and a suggested possibility for minimizing the sources of error points to further investigations of developers for a partial solution of the problem. Numerous photomicrograms well illustrate the mechanism involved. While the interaction of adjacent images may introduce an error of 2 to 3 microns, and is of much concern to spectroscopists, the fact that errors due to general distortion of the film are well below the errors of measurement is the consoling factor for investigators of parallax and proper motion.

An extensive Bibliography, an Index of Authors, and an Index of Subjects complete the volume, which altogether is one of the most im-

portant monographs for the research worker that has appeared in recent years.

The large type, clear reproductions, and generally attractive makeup add to the usefulness of the volume as a ready handbook for all investigators whose fields of study involve the measurement and interpretation of the photographic image.

H. T. STETSON

Einführung in das Studium der Veränderlichen Sterne. By KARL SCHILLER. Leipzig: J. A. Barth, 1923. Pp. viii+384. 45 figures. $3.30.

The purpose of this volume, as stated by the author in his Preface, is to present a comprehensive and at the same time condensed treatment of the topics connected with the study of variable stars. It is intended for those who take up the subject either as amateurs or as specialists. It requires a knowledge of science in general and makes free use of mathematical symbols. It can be commended throughout for its careful and thorough handling of the matter in hand. There are numerous footnotes, the style is clear, and the German is easily read.

Dr. Schiller pays particular attention to the astrophysical principles which underlie the methods of observation or explain the phenomena of variables, and almost half of the book is devoted to the first two sections, "Astrophysical Principles" and "Practical Photometry." The other sections are "The Treatment of Variable Stars," "The Theories of Light Variation," "The Theory of the Eclipsing Variable," and "Systematic Studies."

The first section contains four chapters, the first of which treats of the classification of variables, giving Pickering's well-known arrangement with its most recent subdivisions. This is followed by a discussion of the principles of photometry, including general photometric relations, Fechner's law, Pogson's rule, and the Purkinje phenomenon, with advice to the observer. The author advises especially that a student should test his eyes thoroughly before taking up the observation of variable stars, and gives a list of test objects for the purpose.

The third chapter of this section is devoted to the principles of spectrum analysis, with the classifications of stellar spectra. This topic is especially well done, because it includes the classifications of Secchi, Vogel, that of Harvard, which he calls the Pickering-Cannon classification, and Miss Maury's, with a description of her subdivisions (*a*), (*b*), and (*c*),

which have since become so significant. This is followed by the Lockyer-Hertzsprung-Russell theory of stellar evolution. Doppler's principle ends the chapter.

The color of the stars is the general topic of the fourth chapter, which ends with a table showing the relation between spectral type, color, color-index, effective wave-length, minimum wave-length, and effective temperature.

The second section of the book, which is devoted to practical photometry, consists of six chapters, which treat of the following subjects: eye estimations made without measuring apparatus; visual photometry including the use of shutters, polarizing apparatus, and the wedge; photographic photometry, including the measurement of the diameter of the star-image as well as the density of extra-focal images; the photo-electric cell; a general comparison of the different methods; and extinction.

Both of these sections show very careful handling on the part of the author. They may be criticized for a lack of reference to recent American work on the subject, but he states in his Preface that it was difficult for him to get our literature in Germany.

The third section of the book treats of the variable itself and includes chapters on nomenclature, star charts, catalogues of variables, ephemerides, and charts especially for variables. Other chapters give in detail the methods of handling the observations so as to get the light-curve and the determination of the elements of the light-variation.

Under the title "Theories of Light Variation" the author treats of the temporary stars, of long-period variables, and of the "Blink" stars, giving very good statements of the various theories which have been presented in explanation of the last type.

In treating of the eclipsing variable he gives very explicit directions for determining the orbital elements from the observations, with special reference to the work of Russell, including a numerical example worked out in detail.

The sixth and last section of the book treats of statistical material, and several different combinations of data regarding variables are conveniently arranged in tabular form. An Appendix follows which contains tables for use in getting out the elements of eclipsing variables.

The volume cannot fail to be useful to any student of variable stars, and is most heartily recommended by the reviewer.

CAROLINE E. FURNESS

The Earth, Its Origin, History and Physical Constitution. By HAROLD JEFFREYS. Cambridge: University Press, 1924. Pp. ix+278. 16*s.*

Those readers who may be looking for an encyclopedic manual of geophysics will not find it in this book. On the other hand, those who have followed the author's papers, as they have appeared from time to time in scientific journals, will find that many of the pages have a familiar look. This is not said by way of disparagement. Even if the book were nothing but a reprint of the author's geophysical papers, these would well serve re-publication. But the book is much more than this. By arranging, revising, expanding, and adding, the author has given his work a special unity of its own, the unity of a consecutive treatment of the earth's origin and history, thus connecting mathematically the present with the past. This is perhaps a venturesome undertaking, but one that must sometime be attempted if progress is to be made, and probably there is no one better fitted for the task than the author. Even a fuller summary of the contents than can be given here would do the book less than justice. In the following paragraphs only the barest outline of the general course of the discussion is attempted, and many essential details are omitted.

The first two chapters deal with the origin of the solar system. Laplace's cosmogony is discussed in chapter i and then set aside as inadequate. In chapter ii the author explains his own conception. It involves the close approach to the sun of a passing star and the consequent ejection of matter from the sun to form the planets. In some respects this scheme resembles the planetesimal cosmogony of Chamberlin and Moulton, but it resembles still more closely Jeans's conception of what took place, the chief difference being in the supposed size of the sun when the planetary matter was ejected. The planetesimal hypothesis and Jeans's theory are set forth and discussed in Appendixes.

In chapter iii, Jeffreys accepts as probable Darwin's idea that the moon once formed a part of the earth but was torn away by solar tidal action magnified by the near coincidence of the period of the solar tide with the free period of the earth's vibration. The discussion is simplified by the free use of analogy between the comparatively simple two-dimensional case and the far more complicated three-dimensional one. The heterogeneity of the earth is favorable to the theory, which would be unacceptable if the earth were homogeneous.

Chapter iv deals with the resisting medium composed of the gaseous matter ejected from the sun or escaped from the planets after their formation. This resisting medium would in time either be scattered to infinity

or be gathered into the sun, but meanwhile it would have rotated around the sun with velocities depending on the distance and would have tended to reduce the eccentricities of the planetary orbits. From considerations particularly applicable to Mercury it is estimated that the age of the solar system might range anywhere from 10^9 to 10^{16} years.

Chapter v considers the age of the earth, the chief reliance being on methods based on radioactivity. The resulting age is of the order of 10^9 years.

In chapter vi the thermal history of the earth is discussed. In the partial differential equations for the flow of heat a term is included in order to allow for the heat generated by radioactivity. For reasons not clearly known, this seems to be greatest at the surface and to diminish rapidly with the depth. Two extreme assumptions are made as to the way in which the radioactivity dies out with increasing depth, but even at moderate depths both assumptions give nearly the same thermal history. At a depth of 700 kilometers beneath the continents the temperature is but little changed from what it was when solidification set in, namely, about 1300° C. Beneath the oceans the cooling is rather greater than beneath the continents. At depths of from 200 to 400 kilometers a tendency toward fluidity due to high temperature may be expected to appear.

Chapter vii is merely a mathematical preliminary to chapter viii, which treats of the deformation of the crust under the load of mountains. The treatment is in some ways more comprehensive than Darwin's treatment of this theme, but on the other hand, the curvature of the earth and the gravitational effects of the load are disregarded, so that the discussion would not apply to loads of continental dimensions.

Chapter ix deals with isostasy. The first part is entirely non-mathematical and consists of an interesting, important discussion of the notions of solidity and fluidity and of the behavior of matter under stress. Isostasy, under deposition or other addition of matter, is conceived as arising from the bending of the solid crust under the load, and the consequent displacing of quasi-fluid matter below.

Chapter x presents the theory of mountain formation by differential thermal shrinkage and the consequent wrinkling of the crust. The heat due to radioactivity in the upper crust helps to prevent it from shrinking as fast as the layers below and thus assists in producing wrinkles. Estimates are made of the amount of wrinkling actually present in existing mountain folds, and of the amount that differential shrinkage would explain, and a fair agreement is found between the two results.

Chapter xi discusses the origin of other features of the earth's surface.

Contraction being greater under the cold water of the ocean depths than under the continents, the shrinking crust tends to buckle like a wet book cover when it is dried. This buckling, beginning rather abruptly near the seacoast, produces the marginal deeps so characteristic of the Pacific Ocean. The buckling is sustained by the greater strength of the upper crust beneath the oceans as compared with that beneath the land. This would, of course, imply a departure from perfect local isostasy of ocean basins. Other topics treated are the formation of geosynclines, stresses in a cooling earth, the origin of lunar maria and of terrestrial continents.

Chapter xii explains how knowledge of the earth's interior may be derived from earthquake data. The evidence, especially that from the great explosion of September 21, 1921, at Oppau in the Bavarian Palatinate, indicates that over the continents there is beneath the sedimentary rocks a granitic layer 10 to 20 kilometers thick, and beneath this comes basic rock of various kinds. The velocities deduced from the transmission times of earthquake waves correspond to the elastic properties of these basic rocks rather than to those of the strata above them. Knott's investigation of the velocities of seismic waves in the interior of the earth is reproduced in substance, and the fact is brought out that the distortional waves appear unable to penetrate the central regions of the earth, but—apart from the transitions from sedimentary to granitic and to basic rocks just mentioned[1]—nothing is said about those numerous and sharply defined surfaces of discontinuity within the earth which play such a prominent part in the thinking of many seismologists on the Continent.

Chapter xiii discusses the figures of the earth and moon. The discussion of the form of a nearly spherical, gravitating and rotating body follows familiar lines, but Jeffreys also sets forth the less well-known transformation of Radau, which leads to the conclusion that geodesy and astronomy cannot help us in choosing among those various laws, such as Roche's, Laplace's, or Wiechert's, that connect the density with the distance from the center. The figure of the moon, as deduced dynamically from the inclination of its equator and its libration in longitude, is not consistent with a state of fluid equilibrium at its present distance, but could be explained by supposing that the moon solidified into present form when its distance from the earth was only about three-eighths of its present value. Such departures from fluid equilibrium as this supposition would imply could more easily be maintained on the moon than on the earth, owing to the more complete cooling of the moon and the smaller force of gravity there.

[1] The discontinuity postulated in Wiechert's theory of the constitution of the earth is touched on in the next chapter.

Chapter xiv reproduces Jeffreys' brilliant investigations of tidal friction and shows that oceanic tidal friction is almost entirely confined to shallow seas and is apparently quite adequate by itself to account for the otherwise unexplained portion of apparent secular acceleration of the moon. This would mean that bodily tidal friction in the earth must be very small. The chapter contains also a prophecy of the future relations of the earth and moon and some discussion of tidal friction on other planets and satellites.

Chapter xv treats of the variation of latitude. The existence of the fourteen-month term in the variation is a further proof that bodily tidal friction—of the elastico-viscous sort—cannot be the main cause of the apparent secular acceleration of the moon. Internal friction of the ordinary type great enough to account for the entire apparent secular acceleration of the moon would very promptly damp out the fourteen-month oscillation of the poles.

Besides the Appendixes on cosmogonic theories, already mentioned, there is another Appendix containing some further discussion of the behavior of matter under stress, with especial reference to the assumption sometimes made that the matter in the earth, and even in the outer crust, may be deformed indefinitely, given time enough, by any stress, however small. This idea is one of the mainstays of the Wegener hypothesis of continental migration. There are two other Appendixes, one on climatic variation and one on empirical periodicities.

A work covering so many and such highly speculative and controversial subjects will inevitably give rise to many objections. T. C. Chamberlin[1] has already questioned Jeffreys' treatment of the planetesimal hypothesis, and R. T. Chamberlin[2] has questioned Jeffreys' estimate of the actual shortening of the earth's crust as revealed by existing mountain folds. Perhaps Eddington's recent work on stellar physics may bring about some changes in the author's ideas regarding the early history of the solar system. The reviewer might add a few criticisms more nearly in his own field of work. Among these are:

1. Chapter ix. If the free-air method of reducing gravity to sea-level is to have the name of any person connected with it, that name should be Faye rather than Helmert.

2. Chapter ix. On the other hand, the use of the free-air method by Helmert, or any one else, does not necessarily imply the acceptance of any theory about isostasy or the interior of the earth. The method can be justified without any such assumption.

[1] *Journal of Geology*, **32**, 696, 1924.

[2] *Ibid.*, **32**, 550, 1924.

3. Chapter ix. Jeffreys does not discuss or even mention the longitude term in Helmert's formula for theoretical gravity, although, for another purpose, he refers to the paper where Helmert derives and emphasizes this term. It seems to the reviewer that the existence of Helmert's term might be an argument in favor of some of Jeffreys' ideas about the difference between the crust beneath the oceans and that beneath the continents (chapter xi).

4. Chapter xiv. The assumption of a constant rate of retardation of the earth's rotation by tidal friction is probably made because it would be practically impossible to estimate the rate for some other distribution of land and water. The known existence in past geologic time of extensive areas of shallow sea would, however, seem to indicate that the rate of retardation was formerly much greater.

5. Perhaps an exaggerated importance is attributed to radioactivity. At any rate, investigations of hot springs show that the amount of radioactive matter has little effect on the temperature of the water and no apparent correlation with it.[1]

This, however, is not the place for an extended discussion of mooted questions. Apart from the fact already mentioned that details are consistently discussed in connection with a theory of the earth's origin and history, the outstanding merit of the book is, to the reviewer's mind, the fact that hypotheses are consistently tested by calculation. In a subject like geophysics, where direct experimentation is often impossible, the conclusions must usually be reached deductively, and too often in the past the wildest speculation has gone unchecked by any attempt to estimate quantitatively the consequences of a hypothesis. Some of the so-called explanations of the migration of continents postulated by Wegener might be cited as awful examples of speculation uncontrolled by calculation. At present, however, even the knowledge of an order of magnitude of a quantity is sometimes about as satisfactory as the result of more precise—and much more laborious—calculation. The author shows great ingenuity in estimating orders of magnitude by comparatively simple means, though sometimes the simplicity may be more apparent than real to those readers —necessarily an overwhelming majority—whose mathematical skill is considerably less than the author's. In fact, in the Preface the author remarks that even those mathematically trained will often be tempted to skip the mathematical analysis. Those who can do so, however, will find it well worth while to follow the argument in detail, and it is only from the labors of those who can and will do so critically that further progress

[1] *Journal of Geology*, **32**, 468, 1924 (Symposium on Hot Springs).

is to be expected. Moreover, there is much matter that will be found interesting and profitable even to the non-mathematical student. The book is one of the outstanding achievements in geophysics of the year 1924.

WALTER D. LAMBERT

The Atom and the Bohr Theory of Its Structure: An Elementary Presentation. By H. A. KRAMERS and HELGE HOLST. Translated from the Danish by R. B. and R. T. LINDSAY. New York: Alfred A. Knopf, 1923. Pp. xiii+210. Figs. 34, portrait, pls. 2. $4.00 net.

Although originally intended as a semipopular account, this book should prove of great value to all who are interested in modern physical theories. In fact, the portions dealing with the Bohr theory must be regarded as an authoritative statement of the Danish physicist's viewpoint, as Dr. Kramers has been closely associated with him in all the more recent developments of the subject. The treatment of atomic physics in general is concise and shorn of all unessential detail—to such an extent even that to many more sophisticated readers it may appear to present an artificially simplified picture.

Fully half of the book is devoted to a simple and straightforward account of atomic physics and the theory of radiation prior to the appearance in 1913 of Bohr's paper on the explanation of the hydrogen spectrum. No previous knowledge of even the elementary concepts of physics is assumed on the part of the reader, and the treatment is strictly non-mathematical throughout. The second half consists of an exposition in equally simple language of the fundamental ideas of the Bohr theory and their application to radiation phenomena, concluding with a chapter on atomic structure and chemical properties. Considerable space is given to a discussion of the character of the explanation which the new theory gives of the phenomena which are so troublesome when the classical theory is applied to the nuclear atom; and ideas involved in the correspondence principle, as well as the delicate question of light quanta versus wave theory, are given an extremely simplified but fundamental treatment. This portion of the book should prove of particular interest to all students of the quantum theory.

FRANK C. HOYT

JOHN ADELBERT PARKHURST

It is with deep regret that we record the sudden death on March 1, 1925, in his sixty-fourth year, of John Adelbert Parkhurst, Associate Professor of Practical Astronomy in the University of Chicago and member of the staff of the Yerkes Observatory since 1900. The editors of this *Journal* have been always able to avail themselves of his excellent judgment in matters concerning his specialty of stellar photometry. He had also furnished us with many reviews of works in this field of astronomy. Our pages have contained about twenty longer articles and an equal number of shorter contributions on his researches. One of the most important of these was entitled "The Yerkes Actinometry," which appeared in October, 1912, and occupied a large part of that issue. It gave his determination of the photovisual and photographic magnitudes, color-indices, and spectral classes of 672 stars, chiefly brighter than the eighth magnitude, between declination $+73^\circ$ and the North Pole.

Mr. Parkhurst collaborated with George E. Hale and Ferdinand Ellerman in the measurement and discussion of the spectra of stars of Secchi's fourth type, which appeared in Volume II of the *Publications of the Yerkes Observatory*.

His *Researches on Stellar Photometry*, of 192 pages, was published in 1906 by the Carnegie Institution of Washington.

He participated in the joint undertaking proposed by the late Professor E. C. Pickering for the extension of the visual photometric scale to faint stars in the so-called "Rumford Fields." This was published in 1923 in the *Memoirs* of the American Academy of Arts and Sciences.

An important investigation by Mr. Parkhurst, to be published later, was the determination of the photographic and photovisual brightness of about 1,500 stars in the 24 "Selected Areas" of Kapteyn in the zone of declination $+45^\circ$. The plates, taken with the 40-inch refractor, 24-inch reflector, and Zeiss U.-V. doublet, had been measured and partially discussed.

The work of this skilful observer and patient investigator has done much to make more precise the determination of the brightness of the stars.

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ERRATA

Vol. **60,** No. 3, October, 1924, article on: "The System of 61 μ Orionis," by Edwin B. Frost and Otto Struve:

Page 197, Table III, fourth column, in the first, third, fifth, seventh, and ninth lines, *for* 2,423,863 *read* 2,423,862.

Vol. **61**, No. 1, January, 1925, article on: "Joining the Infra-Red and Electric-Wave Spectra," by E. F. Nichols and J. D. Tear:

Page 19, line 2, *for* A. W. Hull *read* G. F. Hull.